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Modeling and mapping of magnetic stars on the base of field sources

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Abstract. Models of magnetic stars are constructed by the method of the magnetic charge distribution (MCD). The surface magnetic field is the linear summation of the vector components of the individual fields of virtual magnetic monopoles, which combine to magnetic dipoles and multipoles. The MCD method relates to the construction of a vector field out of its sources and vortices, which is comfortable for progamming on a computer and possesses a wide range of applicability. The modeling is realized for the straightforward calculation by a computer program, which fits the calculated phase curves to the observed ones by variation of parameters and iterative least squares optimization. The magnetic map of the star is drawn for the surface field accounting for all four Stokes parameters. From the map the phase curves and the (geometrically caused) profiles of the spectral lines are derived.

Key words: stars: chemically peculiar – stars: magnetic fields – stars: rotation – stars: modeling – stars: mapping

1. Introduction

The distribution of the magnetic field of a star over its surface is covered from observation by many information deforming processes. For the reconstruction of the original surface distribution from the final observational values all these processes have to be inverted. The difficulties bound to the generally ill-posed inverse problem are well-known and have been considered by Khokhlova et al. (1986).

In contrary to this a straightforward calculation can be carried out in any case. Assuming physically reasonable conditions we use only a few parameters to construct a magnetic map at the surface of the star, which will be improved by iteration.

At first a **model** will be constructed using reasonable but, nevertheless, arbitrary parameters. For this task Bagnulo et al. (1996, 1998, 1999) have developed independently of ours a method using for the calculation of the surface field spherical harmonics. The field strengths at the magnetic poles are taken as the parameters.

On the contrary, we calculate a polar magnetic field strength and use **magnetic charges** as initial parameters. All these minimize the parameter set. The MCD method has been described (Gerth et al. 1997; Gerth, Glagolevskij 2001) and applied (Gerth et al. 1998, 1999; Gerth, Glagolevskij 2001; Glagolevskij et al. 1998a,b,c) by the authors. Khalack et al. (2001, 2002, 2003) also used this method. This paper is restricted to the theoretical foundation of the method in order to give an additional information about the theoretical magnitude background.

2. Theory and calculation of stellar magnetic fields

The calculation of magnetic fields in stars has an old history. The first who used spherical harmonics for the formulation of the magnetic field structure on the surface of a star was A. J. Deutsch (1970). We refer here especially to the papers of Oetken (1977, 1979), who modelled the star as an equatorially symmetric rotator. Oetken relates to Krause and Rädler (1980), who attempted to calculate the magnetic field structure of a star as generated by the action of a dynamo.

The solution of the **hydromagnetic differential equations** of the dynamo is displayed as a series of Legendre functions. However, although the mathematical treatment using Legendre polynomials yields an analytical function for the surface field and a fit to observations, it conceals the physical meaning of the coefficients and the origin of the magnetic field. Of special interest are, obviously, the eigenvalues as the solution of the Legendre differential equation, which we acknowledge as the intrinsic source of the magnetic field.

The **magnetic field**, of course, originates in the interior of the star and penetrates the spherical surface of the star's atmosphere. Only from this location the magnetic field can be observed by the Zeeman displacement of spectral lines. The mapping of the surface field is the two-dimensional cartographic arrangement of the magnetic features of the outermost layer of the star. Any assumption about structure of the inner magnetic field on account of surface field data demands physical basing. In any case we have to take the whole spatial distribution of the field into account. The magnetic field itself is a **vector field**, which is defined completely by its sources and vortices. If we know the magnitudes and the spatial locations of them we can calculate the components of the field vector in any point of the surrounding space.

Sources and vortices constitute different kinds of field generators. Sources exist as individual monopoles, from which the field lines diverge radially, whereas whirls are circulated by closed field rings with a left or right handed rotation around an axial vector. The interaction of both sources and vortices is governed by Maxwell's equations and Ohm's law (Rädler 1995), which gives the condition for the excitation of a dynamo in the electro-conducting turbulent medium of the star. This is well established for the Sun.

The magnetic stars, which we investigate, are mainly A stars. They show a very quiet behavior without turbulence, so that a dynamo mechanism cannot act. Obviously, we have to deal with a **long-living permanent magnetism**.

We restrict the following considerations of the magnetic field structures in stars only to the stationary state, which is relevant for mapping. In stationary conditions sources or vortices can exist separately.

According to Maxwell's laws, the stationary magnetic field is represented only by the vortices. Indeed, the construction of the magnetic field by vortices is possible but rather complicated for conception and calculation. The spherical field of a point-like source is much more better suited for the calculation. For a real magnetic field, the field vector B in absence of sources is expressed by the relation

$$\mathrm{div}\boldsymbol{B} = 0. \qquad (1)$$

This means, that the magnetic field lines are closed and have neither a beginning nor an ending point. Thus, magnetic monopoles do not exist in reality. However, such a magnetic dipole is self-consisting like an electric dipole of two oppositely signed charges. We formally call a magnetic field a dipole one because of its components behave like components of electric dipole field. This magnetic field is such as if created by two virtual magnetic charges. In Fig. 1 the magnetic field structure of a circulating electric current is compared with the dipole field of two magnetic charges of opposite polarity.

The magnetic dipole is a real physical quantity with a magnetic moment

$$\boldsymbol{M}=Q\boldsymbol{l}, \qquad (2)$$

where Q is the "magnetic charge" and l is the length difference of the dipole center between the two charge locations. Thus the magnetic moment is a vector and undergoes all rules of vector algebra. This has the following consequences:

1. The magnetic moment produces a magnetic field environment of dipole structure.
2. Spatial vector fields of dipoles superpose by vector addition.
3. The sum of several magnetic moments at the same location yields a resultant magnetic moment, maintaining its environmental dipole structure.
4. The length $2\boldsymbol{l}$ spanning the distance between the virtual magnetic charges is an infinitesimal quantity $\boldsymbol{l} \to 0$ for the mathematical dipole, but it can take on real significance for separated charges as realized in form of a rod magnet.
5. The virtual magnetic charges of dipoles and multipoles may formally be treated like separated individual field sources with arbitrary spatial distribution — provided the coupling of pairs with opposite sign and the sum of all charges Q_i being zero according to (1) is preserved:

$$\sum_i Q_i = 0 \qquad (3)$$

Exploiting the numerical advantages of the point-like sources, we should not abandon at all the calculation of the magnetic field by vortices. We will do this for completeness. In practice, however, we relate to the comfortable magnetic charges.

3. The physical significance of magnetic charges

The treatment of the *magnetic charges* as individual and separated field sources renders an important advantage because the arrangement of the locations in the star's body becomes very simple: each location of a charge is determined by 3 spatial coordinates. Fig. 2 illustrates the arrangements of the location of a source and the coordinated field vectors in Cartesian and spherical coordinates.

The magnetic moment of the dipole as a vector is defined by 2 point locations or 6 coordinate values. There is no restriction to a mathematical dipole (with $l \to 0$) or to any spherical or axial symmetry. Dipoles and multipoles might be decentered anyhow. Even sunspots as narrowly located sources under the Sun's surface may be described easily.

Equation (2) is derived for a magnetic dipole. However, it might be understood also as the magnetic moment $\boldsymbol{M}_i$ of a single charge Q_i in the distance $\boldsymbol{l}_i$

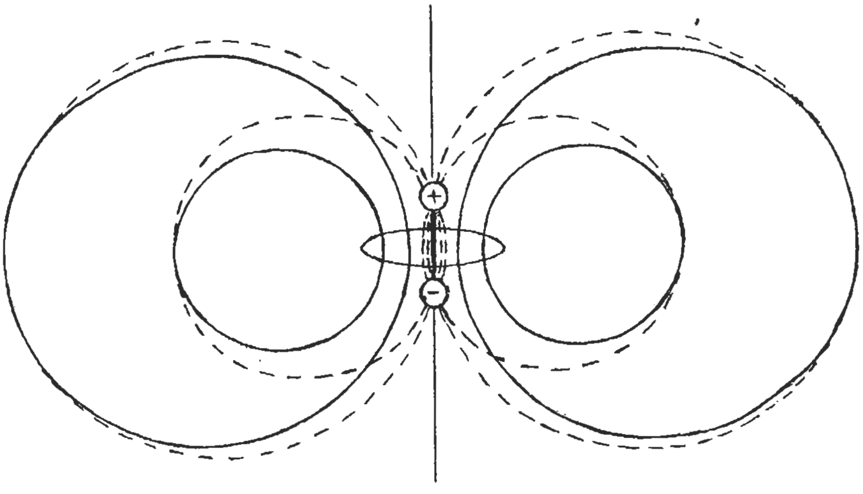

Figure 1: *Demonstration of a dipole with virtual sources.*
An electric current flowing through a loop produces a magnetic field configuration with closed lines of force (solid lines), which penetrate the plane of the loop without crossing. The diverging lines of force (dotted lines) can be traced back to their origin as if it was a dipole with virtual sources – in analogy to virtual images in optics. The approximation of a dipole with virtual sources is the better, the more the quotient R/r increases.

from the center of the sphere. By this way the advantage of the spatial arrangement of magnetic charges is preserved. Then the magnetic dipole moment $\boldsymbol{M}_d$ is the vector sum

$$\boldsymbol{M}_d = Q_1 \boldsymbol{l}_1 + Q_2 \boldsymbol{l}_2 \quad \text{with } Q_2 = -\, Q_1. \tag{4}$$

But now we have to distinguish between poles and charges. The field strength at the poles, which we reduce from observation, is not a primary magnitude but only a derived one. The primary magnitude is the magnetic moment $\boldsymbol{M} = \mathrm{Q}\boldsymbol{l}$, from which all other magnitudes of the magnetic field are derived. These magnitudes have often been confused, so that the physical dimensions of them should be born in mind.

The magnetic field strength $\boldsymbol{B} = \mu \boldsymbol{H}$ ($\boldsymbol{B}$ — magnetic induction, $\boldsymbol{H}$ — magnetic field strength, μ — magnetic permeability) is usually measured by astronomers in gauss. Not differing from this habit, the magnetic charge at the center of a sphere of radius R with the field strength $\boldsymbol{B}$ at the surface is

$$\mathrm{Q} = 4\pi R^2 |\boldsymbol{B}|. \tag{5}$$

Then the physical dimension of the magnetic charge is

field strength × surface area
(or in units: *gauss*$\cdot m^2$).

Likewise, the dimension of the magnetic moment is

field strength × volume
(or in units: *gauss* $\cdot\, m^3$):

$$|M| = \frac{4}{3}\pi R^3 |\boldsymbol{B}|. \tag{6}$$

The magnetic charge produces a central symmetric potential U at the surface of the sphere of the radius R:

$$U = \frac{Q}{4\pi R}. \tag{7}$$

If the charge is displaced from the center of the star, the polar coordinates (radius r, longitude φ, latitude δ) determine its point of location. Then by transformation to Cartesian coordinates

$$\begin{aligned} x &= r\, \cos\delta \cos\varphi, \\ y &= r \cos\delta \sin\varphi, \\ z &= r\, \sin\delta, \end{aligned} \tag{8}$$

we have with $a = r/R$ as the fraction of the star's radius the distance from the center to the i-th point of the source

$$\begin{aligned} r^2 = R^2[&(\cos\delta\cos\varphi - a_i \cos\delta_i \cos\varphi_i)^2 + \\ &+ (\cos\delta \sin\varphi - a_i \cos\delta_i \sin\varphi_i)^2 + \\ &+ (\sin\delta - a_i \sin\delta_i)^2]. \end{aligned} \tag{9}$$

The potential of the i-th charge is

$$U_i = \frac{Q_i}{4\pi r_i}. \tag{10}$$

The potentials of several charges are superposed linearly:

$$U = \sum_i U_i. \tag{11}$$

Especially the potential U_d of a dipole with the charge

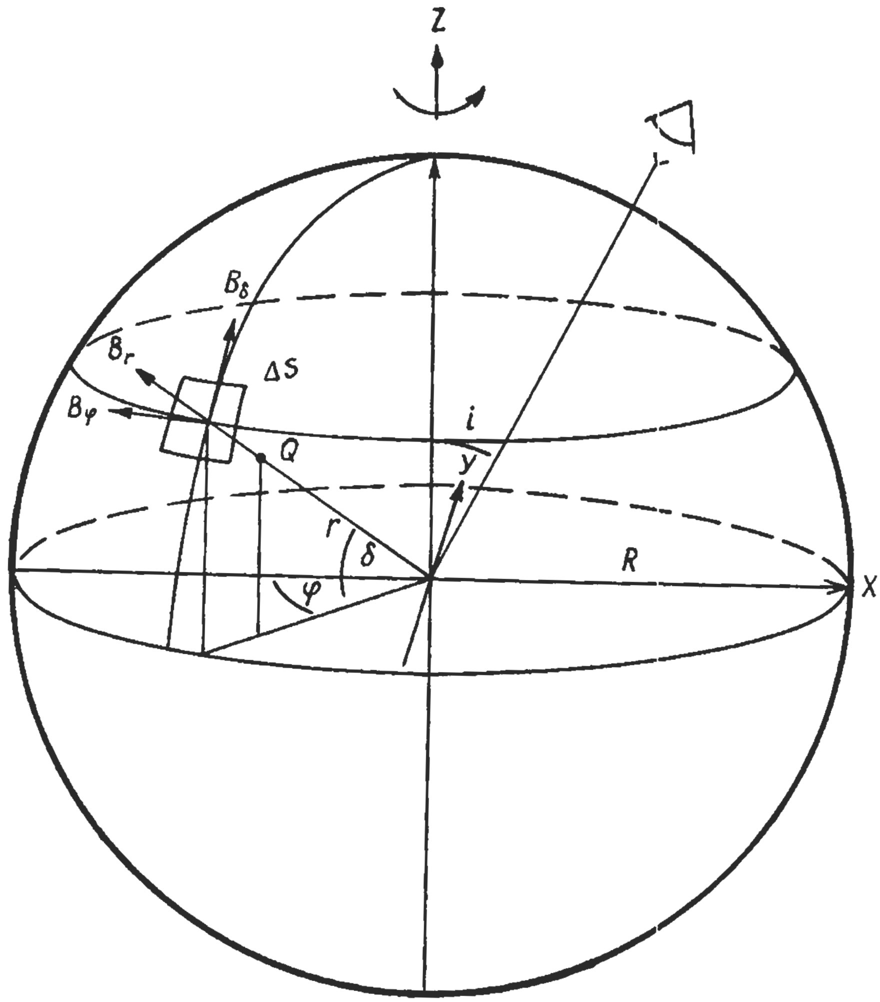

Figure 2: *Geometry of a point-like source in a sphere.*
The star is orientated in the Cartesian coordinate system with its rotation axis coinciding with the z-coordinate, at which the observer looks by the inclination angle i. The origin point of the field is located in spherical coordinates φ *(longitude),* δ *(latitude), r (radius-fraction), corresponding to equations (8), in the distance r/R from the center of the sphere with radius R.*
The three orthogonal components $\boldsymbol{B}_r$, $\boldsymbol{B}_\varphi$, $\boldsymbol{B}_\delta$ *of the field vector in the center of the surface element* $\triangle S$ *are given by equations (18).*

Q and r_+, r_- for each source is given by

$$U_d = \frac{Q}{4\pi(1/r_+ - 1/r_-)}. \tag{12}$$

4. Construction of a potential field of a magnetic charge

From the scalar potential U the field strength is derived by the linear differential operator gradient

$$\boldsymbol{B} = -\mathrm{grad}U. \tag{13}$$

The gradient is a vector of 3 components, which span a space with 3 orthogonal unity vectors as Cartesian or spherical coordinates.

In Cartesian coordinates x, y, z we have with the unity vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ the gradient

$$\mathrm{grad}U = \frac{\partial U}{\partial x}\mathbf{i} + \frac{\partial U}{\partial y}\mathbf{j} + \frac{\partial U}{\partial z}\mathbf{k}. \tag{14}$$

Likewise, we have for each point of the sphere in the polar orthogonal system of radius r, longitude φ and latitude δ the gradient

$$\mathrm{grad}U = \frac{\partial U}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}x}\mathbf{i} + \frac{\partial U}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}y}\mathbf{j} + \frac{\partial U}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}z}\mathbf{k}. \tag{15}$$

If we consider only the one-dimensional case using for the polar coordinates the radius r and simplify the constant with the charge Q to $C = -\frac{Q}{4\pi}$, then the potential

$$U = -\frac{C}{r} \quad \text{yields the gradient} \tag{16}$$

$$\frac{\mathrm{d}U}{\mathrm{d}r} = \frac{C}{r^2} \ . \tag{17}$$

The differential quotients, that give the gradient along the 3 orthogonal polar coordinates, are:

$$\begin{aligned} \boldsymbol{B}_r &= \partial U/\partial r = (C/r^3)[\cos\delta(\cos\varphi + \sin\varphi) + \sin\delta] \\ \boldsymbol{B}_\varphi &= \partial U/\partial\varphi = (aC/r^3)\cos\delta(\cos\varphi - \sin\varphi) \qquad (18) \\ \boldsymbol{B}_\delta &= \partial U/\partial\delta = (aC/r^3)[\cos\delta - \sin\delta(\sin\varphi + \cos\varphi)] \end{aligned}$$

These equations are the basic relations for the calculation of the magnetic field strength distribution over the star's surface for a single monopole. The differential quotients represent the 3 coordinates of the magnetic field at the surface of the star, which constitute the field vector. The mapping of the magnetic surface structure relates to these values.

5. Construction of a magnetic vortex field

A real magnetic field is a combination of different fields of dipoles and vortices, which superpose linearly. Like the gradient for the magnetic dipole, the calculation of the field strength for the magnetic vortex is based on the linear differential operator **curl**.

A vortex constitutes the closed magnetic lines of force around an axial vector with origin at spherical coordinates r, φ, δ and direction determined by the spatial motion of an electrical charge through Cartesian space. The three vector components of the electrical current $\boldsymbol{I}$, with origin at Cartesian coordinates x, y, z on the sphere with radius r, can be written in spherical coordinates also with three parameters: the magnitude of the current $\boldsymbol{I}$, and λ, the horizontal component and ϑ, the azimuthal component.

The field strength of a vortex is derived by the vectorial differential operator:

$$\mathrm{curl}\boldsymbol{I} = \left(\frac{\partial I_z}{\partial y} - \frac{\partial I_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial I_x}{\partial z} - \frac{\partial I_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y}\right)\mathbf{k} \tag{19}$$

The partial differential quotients of the Cartesian components of the current, I_x, I_y, I_z, are calculated in the same manner as the differential quotients of the potential U corresponding to equation (5) with terms like equation (18).

6. Algorithms for sources, vortices, and spherical functions

Stationary fields can be calculated only by means of dipole fields. Therefore, the algorithm of the gradient is the most important one that we use for the construction of stellar magnetic fields. In the case of a dipole, a superposition of two monopole fields of opposite sign takes place. Therefore, the algorithm of the monopole is run twice, summing the field values corresponding to the surface coordinates in the element boxes. The summation of the fields of monopoles can arbitrarily be continued. For magnetic fields the pair-like combination of magnetic charges has to be obeyed, because the sum of the charges must be zero.

The differential operators grad and curl have been programmed for a computer as standard algorithms, which are embedded in the framework of an entire complex program for the computation of the magnetic structure in the surrounding space. It should be emphasized, that the structure of the magnetic field on the surface of the star's sphere for magnetic mapping is only a special case of the field on a spherical plane in the spatial field distribution. In this respect the MCD method is more general than the computation of the surface field by spherical harmonics (Legendre functions).

In order to compare the methods, the authors have programmed also the computation of the surface field on the base of spherical harmonics. The analytical formulation we have got courtesy to Professor Rädler from the Astrophysical Institute Potsdam. The algorithm for Legendre's polynomials P_p^q is general for the degree q and the order index p and can be evaluated up to the limiting accuracy of the computer. It is called in the same program as those of the sources and vortices.

Interestingly, the physical meaning of the coefficients of Legendre's polynomials can easily be determined and thus identified by graphical comparison of the calculated magnetic maps. For the calculation of magnetic fields, it turns out that the coefficients are magnetic moments, which can be expressed by dipoles, quadrupoles etc., i.e., with special arrangements of magnetic charges. The surface of the star is occupied by the magnetic field, which might be calculated by any of these mentioned methods. Rädler's method shows very strikingly, that all magnetic fields can be constructed also on the base of combinations of elementary magnetic dipoles, whose field vectors superpose linearly.

The MCD method resorts only to magnetic monopole charges and uses for the calculation of the spherically symmetric magnetic field in the surrounding space of a point-like source as the elementary algorithm the gradient of the potential. By this method all configurations of a magnetic body with a stationary magnetic field can be constellated completely, rendering the algorithms of the vortex and the spherical harmonics dispensable.

For the practical calculation with a computer the surface has to be divided in $n \cdot 2n$ surface elements, which are arranged as a quadratic matrix with the rank $2n$. Using the normal coordinates, the longitude is divided in $2n$ and latitude — in n parts, corresponding to the cartographical arrangement of a Mercator map. The 3 components of the magnetic field vector are calculated for each individual monopole and then stored in the element boxes of the matrix, from where they may be recalled for further computations. For the superposition of the fields of numerous monopoles (or vortices), the already calculated field values are recalled, then added to the actual values of a next monopole field, and the sum will be restored again. This procedure continues successively, using only the elementary algorithms for each calculation of a monopole, a vortex or a term of the expansion of spherical harmonics.

7. Observation of the integral radiation from the star

The calculation of the magnetic field strength renders a triple of values to every point of the stellar surface. However, the visibility of such a surface point depends on a lot of conditions bound to geometry, phase and physics of the star. In numerical computation such point is the center of an element.

The resultant values of the magnetic field strengths distributed over the stellar surface represent the map of the star $B(\varphi, \delta)$. The globe of the star is seen by the observer under different aspects, caused by its rotation and the inclination i to the rotational axis. Besides of this, the visible disk is "vignetted" (shaded off gradually to the edge like an overlaid vignette) by the limb darkening according to the empirical formula with ε denoting the angle from the center of the disk

$$k = 1 - 0.4 \cos \varepsilon. \tag{20}$$

We mention marginally that this simple formula (20), which is valid approximately for a gray atmosphere, might be replaced by a more serious procedure regarding the radiation transfer process. The transfer function $k(\varepsilon)$ depends also on the wavelength and the polarization.

For the visibility of the star by the observer we define a window function $W(i, \varepsilon, \delta, \varphi)$, containing the inclination i, the projection of each surface element to the line of sight, and the limb darkening with its angular distance ε from the center of the visible disk, which averages and normalizes the magnetic map distribution function $B(\varphi, \delta)$:

$$\boldsymbol{B}_{\text{eff}}(t) = \frac{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} B(\delta, \varphi) W(i, \varepsilon, \delta, \varphi - t) \mathrm{d}\varphi \mathrm{d}\delta}{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} W(i, \varepsilon, \delta, \varphi - t) \mathrm{d}\varphi \mathrm{d}\delta} \tag{21}$$

This is the general relation between the magnetic field map and the observable *integral radiation flux* of any magnitude over the visible stellar surface, which we use especially for the magnetic field with all its vector components. The integral formula gives the integral mean of the disk seen by the observer and comprises the *convolution integral*, which represents the rotation of the star with its map $B(\varphi, \delta)$ behind the window $W(i, \varepsilon, \delta, \varphi)$. The magnitude t characterizes the rotation at the time of the momentary orientation angle as a function of time. This is the integral phase curve of the effective field strength B_e or the mean surface magnetic field B_s. The denominator makes the normalization. For the numerical calculation we replace the integral transformations by matrix multiplication. The map is discretised into surface areas as matrix elements, each element representing the integral mean value of this area.

8. The effective magnetic field $\boldsymbol{B}_e$ and the mean surface field $\boldsymbol{B}_s$

The magnetic field strength, which the observer measures by the *Zeeman* splitting of spectral lines, is the result of

- projection by the coordinate orientation,
- weighting by different areas of the elements

and

- shading by limb-darkening.

The resulting vector of the magnetic field obtained by integration over the visible disk of the star is orientated anyhow, but only the projection to the line of sight to the observer gives the so called "*longitudinal field vector*" $\boldsymbol{B}_e$, which is the average value of the radiation from all visible elements influenced by the above mentioned conditions. Its absolute value is the *mean surface field* B_s as the magnitude of the visible integral field strength. The averaging rises some physical problems which we have to consider in the following.

Usually we measure the (effective) stellar magnetic field from the *Zeeman displacement* of the gravity centers of the line profiles of oppositely circularly polarized light. What we call the "*effective magnetic field*" B_e is not a mean value but already the result of weighting and convolution of the radiation flux containing the magnetic field information about the form and the position of the profiles of all surface elements. In principle, the transfer of the flux through the atmosphere has to be treated correctly by the methods of radiation transfer theory, rendering the spectral dependence of the limb darkening.

In our computing program we relate to the fact, that the gravity center of two profiles of different height and position is given by the mean of the centers weighted by the profile integrals. Thus, we weight the magnetic field vector, projected onto the line of sight, of all surface elements with their spherical projection and limb darkening and integrate them over the visible hemisphere.

The radial direction of the field vector in every element on the surface is given in Cartesian coordinates with the unity vectors **i**, **j**, **k** and the geographical coordinates of the longitude φ and the latitude δ to

$$\mathbf{a}_r = \mathbf{a} = \cos\delta\cos\varphi\mathbf{i} + \cos\delta\sin\varphi\mathbf{j} + \sin\delta\mathbf{k}. \quad (22)$$

The two orthogonal tangential directions to the normal direction follow by partial differentiation:

$$\mathbf{a}_\varphi = \partial\mathbf{a}/\partial\varphi = -\cos\delta\sin\varphi\mathbf{i} + \cos\delta\cos\varphi\mathbf{j}, \quad (23)$$

$$\mathbf{a}_\delta = \partial a/\partial\delta = -\sin\delta\cos\varphi\mathbf{i} - \sin\delta\sin\varphi\mathbf{j} + \cos\delta\mathbf{k} \quad (24)$$

The magnetic field vector at the surface of the star is given in Cartesian coordinates in terms of 3 spherical components B_r, B_φ, and B_δ:

$$\boldsymbol{B} = B_r\mathbf{a}_r + B_\varphi\mathbf{a}_\varphi + B_\delta\mathbf{a}_\delta. \quad (25)$$

The magnetic field components are seen from a special aspect and are projected onto the line of sight, giving the vector which we denote as **v**. For this vector an equation similar to (22) can be obtained if we replace φ by t and δ by $90^o - i$ (i is the inclination angle and t is the rotation position angle):

$$\mathbf{v} = \sin i\cos t\mathbf{i} + \sin t\sin t\mathbf{j} - cos i\mathbf{k}. \quad (26)$$

The projection of the magnetic field vector related to each point of the surface is obtained by a scalar multiplication of the magnetic field vector adjusted to the vector of the line of sight, this is, with the spherical components of the field vector B_a, B_φ, and B_δ, the scalar product

$$B_\mathrm{V} = \boldsymbol{B}\ \mathbf{v} = (B_r\mathbf{a} + B_\varphi\mathbf{a}_\varphi + B_\delta\mathbf{a}_\delta)\mathbf{v}, \quad (27)$$

which yields the scalar field components related to the 3 polar coordinates of the surface elements

$$\begin{aligned} B_\mathrm{V} &= \boldsymbol{B}\ \mathbf{v} = \\ &= B_r[\cos\delta\sin i(\cos\varphi\cos t + \sin\varphi\sin t) - \\ &- \sin\delta\cos i] + \\ &+ B_\varphi[\cos\delta\sin i(\cos\varphi\sin t - \sin\varphi\cos t)] + \\ &+ B_\delta[-\sin\delta\sin i(\cos\varphi\cos t + \sin\varphi\sin t) - \\ &- \cos\delta\cos i]. \end{aligned} \quad (28)$$

This projection of the magnetic field B onto the aspect vector **v** allows the calculation of the longitudinal magnetic field B_e. The components of the vector undergo the averaging by the integral equation (21). The result is the longitudinal magnetic field strength, which we call the "effective" field $\boldsymbol{B}_{\mathrm{eff}}$. This is the magnitude we measure by the Zeeman displacement of the right- and left-handed circularly polarized light. By this way it turns out that equation (28) proves to be the Stokes parameter V, multiplied by the intensity I.

9. The magnetic field in polarized light (Stokes parameters I, Q, U, V,)

Likewise, we can calculate the projection onto the plane perpendicular to the line of sight, if we draw the scalar products of the two orthogonal directions to the vector **v** in equation (26), including all 4 components of the so-called *"Stokes vector"*. Thus, also the linear polarization is respected. The two perpendicular vectors to v are found by permutation of the unity vectors **i**, **j**, **k** :

$$\mathbf{q} = \sin i\sin t\mathbf{i} - \cos i\mathbf{j} + \sin i\cos t\mathbf{k}, \quad (29)$$

$$\mathbf{u} = -\cos i\mathbf{i} + \sin i\sin t\mathbf{j} + \sin i\cos t\mathbf{k}. \quad (30)$$

Thus, the linear field components of the linear polarization, namely the Stokes parameters Q and U, are derived also by scalar multiplication:

$$\begin{aligned} B_\mathrm{q} &= \mathbf{B}\ \mathbf{q} = \\ &= B_r(\cos\delta\cos\varphi\sin i\sin t - \cos\delta\sin\varphi\cos i + \\ &+ \sin\delta\sin i\cos t) + \\ &+ B_\varphi(-\cos\delta\sin\varphi\sin i\sin t - \cos\delta\cos\varphi\cos i) + \\ &+ B_\delta(-\sin\delta\cos\varphi\sin i\sin t + \sin\delta\sin\varphi\cos i + \\ &+ \cos\delta\sin i\cos t), \end{aligned} \quad (31)$$

$$\begin{aligned} B_\mathrm{u} &= \mathbf{B}\ \mathbf{u} = \\ &= B_r(-\cos\delta\cos\varphi\cos i + cos\delta\sin\varphi\sin i\sin t + \\ &+ \sin\delta\sin i\cos t) + \\ &+ B_\varphi(\cos\delta\sin\varphi\cos i + cos\delta\cos\varphi\sin i\sin t) + \\ &+ B_\delta(\sin\delta\cos\varphi\cos i - sin\delta\sin\varphi\sin i\sin t + \\ &+ \cos\delta\sin i\cos t). \end{aligned} \quad (32)$$

The components B_q, B_u, and B_v represent three of the 4 Stokes parameters I, Q, U, and V, constituting the *Stokes vector*, which describes the polarization condition of the light as electro-magnetic radiation (Stokes 1852)

$\mathrm{I} = \sqrt{Q^2 + U^2 + V^2}$ –intensity (33)
$\mathrm{Q} = \mathrm{I}\cos 2\beta\cos 2\Theta$ – first linear polarization
$\mathrm{U} = \mathrm{I}\cos 2\beta\sin 2\Theta$ – second linear polarization
$\mathrm{V} = \mathrm{I}\sin 2\beta$ – circular polarization
Θ – position angle of the ellipse;
β – relation of the axes.

The intensity I is the 1-st Stokes parameter, which is derived from the squares of the linearly and circularly polarized components Q, U, and V.

The mean magnetic field strength for a surface element is the square root of the intensity, namely

$$B_{mean} = \sqrt{B_q^2 + B_u^2 + B_v^2}. \quad (34)$$

This formula coincides formally with the surface field Bs, equation (19). However, the results differ significantly, because the weighing, shading and integration relate in the first case to the total field vector but in the second case — to the components separately.

If we take out of equation (34) only the Q and U components,

$$B_{cross} = \sqrt{B_q^2 + B_u^2} \ , \qquad (35)$$

then the plane of the crossed linear polarization perpendicularly to the line of sight comes in view. B_{cross} multiplied with the rotational velocity *vsini* is an observable magnitude by the line profile and is called by Bagnulo et al. (1996, 1999, 2000) the "crossover".

10. The distribution of the magnetic field over the surface (magnetic mapping)

The calculated distribution of the magnetic field over the surface can be represented graphically. Thus a cartographic map of the star is drawn with topographical features of the magnitudes. Areas of the magnitudes are distinguished by colors and/or isolines. The isolines — iso-magnetic lines, contours with equal magnitude of magnetic field strength — are arranged as closed lines around the poles, which mark the most characteristic features of the map.

Mapping of a sphere is always a graphical problem. The plane (rectangular) projection corresponds well to the matrix arrangement of the surface elements. In Fig. 3 we demonstrate the mapping by a plane projection, which gives an overlook of the entire spherical surface of the star but has an extension of the longitude towards the poles. The sphere of the star is better shown in the correct perspective by transforming the coordinates into the orthographic equatorial projection. In this case only one half of the sphere can be seen so that the two opposite hemispheres of 180 longitude difference give all surface information.

11. Surface inhomogeneities

In reality the clear composition of magnetic and velocity fields is disturbed very much by surface inhomogeneities, which could be related to different causes. Important is only the fact, that we measure the fields by the form and the shift of the spectral lines, which belong to special elements distributed over the surface spotty-like. If an area on the star surface does not contain the investigated chemical element, then its magnetic field cannot be reflected in spectral lines of this element. Processes of integration and averaging of magnetic field over the total visible stellar disk smear and hide all local features almost hopelessly.

We call these stars with their strange spottiness chemically peculiar (CP-stars). However, in the case of the magnetic stars we can preconceive something about the distribution of the elements over the surface on the base of the diffusion theory. The magnetic field itself distributes the elements due to magnetic properties of the atoms around the magnetic poles. This means that magnetic and transmittance inhomogeneities in the star are not only formally but also physically connected. The pole region is usually also a region of concentration or depletion of elements, from which the radiation with the spectral information about magnetic field and velocity goes out. This gives the possibility to describe also the element distribution like a cartographic map with only a few parameters: we determine the center of the spot and calculate the surrounding environment by a distribution function. For the first approximation, different mathematical functions are suited, e.g., the rectangular or triangular function, the parabola or the Gauss-function with all combinations, which could simulate the physical spot profile. The procedure of setting spots onto the surface is additive, so that arbitrary transfer features may be constructed. The authors intend to report on this object in a later publication.

The element distribution in the stellar atmosphere acts like a filter, which transmits the spectral information about the magnetic field, contained in the line profiles, more or less. Therefore, we can handle the element distribution like a transfer map covering the radiating surface of the star.

In the program a fourth component is foreseen for the radiative transfer factor, which allows photometric modelling and weighting of the magnetic radiation by multiplication.

12. Decentered dipoles

Landstreet (1970) introduced the model of the decentered dipole for the explanation of the observational fact, that the phase curves of the effective magnetic field (e.g., for 53 Cam) show a flat extremum at one pole and a sharp extremum at the other one. By this way the most probable dipole in the oblique rotator model could preserved without resorting to any multipole structure. Indeed, the phase curves could be explained well by this assumption. Only the physical explanation was not quite satisfactory. Nevertheless, the calculation of the magnetic field by the MCD method may be performed easily, setting only the positive and the negative magnetic charges at the location to which the dipole is shifted from the center.

A striking example for a magnetic dipole extremely removed from the center is a sunspot as we demonstrate in Fig. 6. We take the umbra as the exit

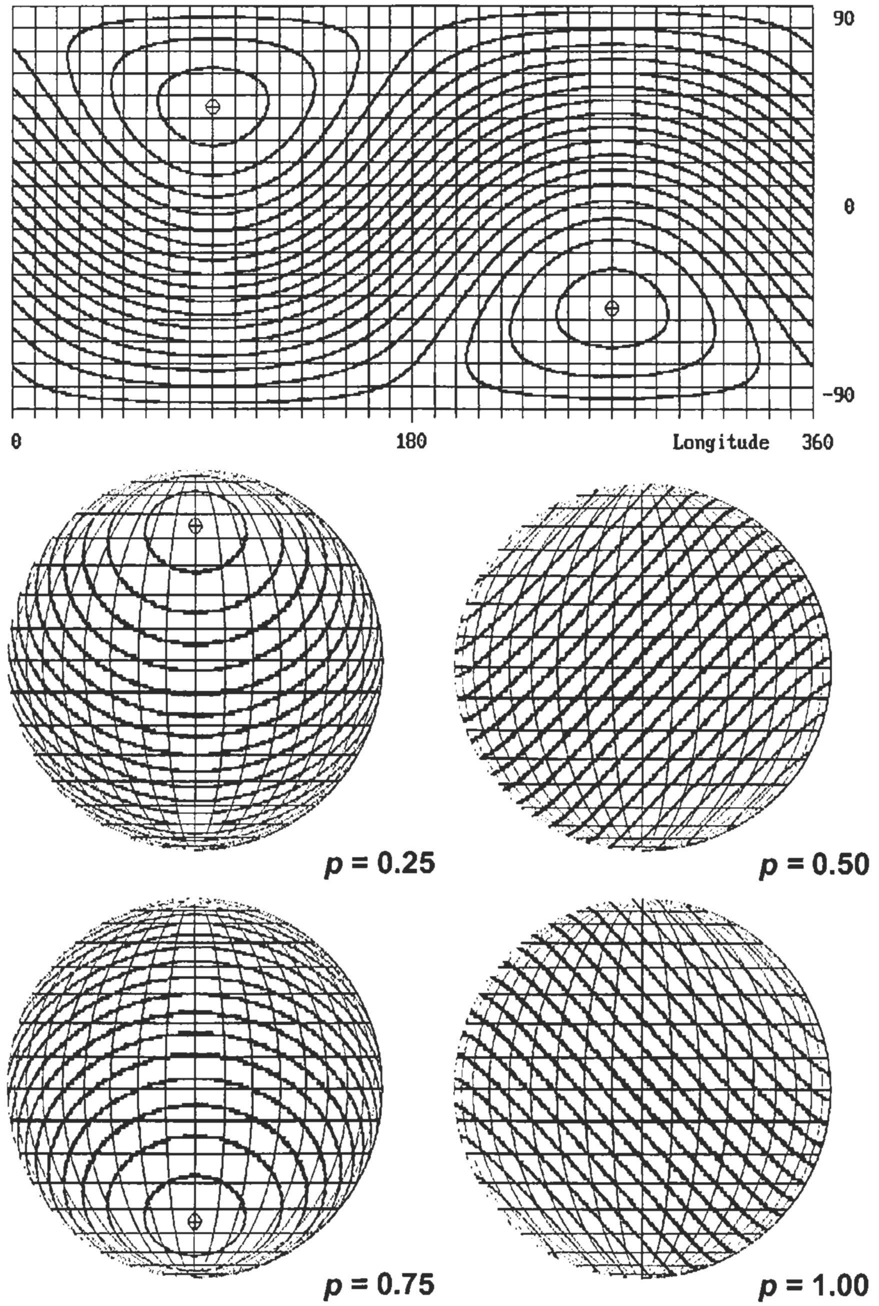

Figure 3: *Mercator map of the magnetic field with the surface elements arranged as a matrix.*
Parameters:

Charge	*Longitude*	*Latitude*	*Radius-fraction*
$Q_1 = +1$	$\varphi_1 = 90^o$	$\delta_1 = +45^o$	$r_1 = 0.1$
$Q_2 = -1$	$\varphi_2 = 270^o$	$\delta_2 = -45^o$	$r_2 = 0.1$

The magnetic charge Q and the radius r are given in relative units. Spherical representation of the map with the phases 0.25, 0.5, 0.75, and 1.00.

of the magnetic field generated by a magnetic charge Q under the surface and locate in some distance l from it a second one of opposite polarity, then a lying on the surface magnetic dipole corresponding to equation (2) is constructed, whose field may be calculated by the program using the MCD method (Gerth & Glagolevskij 2000).

The magnetic source can be even remotely decentered as in the case that the source is located outside the star. Then the main star of a binary system

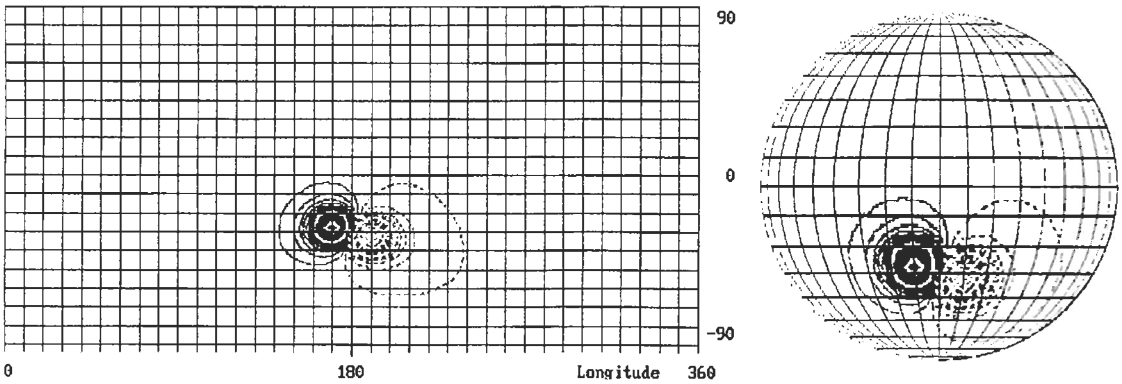

Figure 4: *Mercator-map globe of the global magnetic surface field of a solar-like spot as an example of an extremely decentered magnetic dipole.*
Solid lines — positive region,
dotted lines — negative region.
Parameters:

Charge	*Longitude*	*Latitude*	*Radius-fraction*
$Q_1 = +1$	$\varphi_1 = 170^o$	$\delta_1 = 27.5^o$	$r_1 = 0.88$
$Q_2 = -1$	$\varphi_2 = 190^o$	$\delta_2 = 32.5^o$	$r_2 = 0.92$

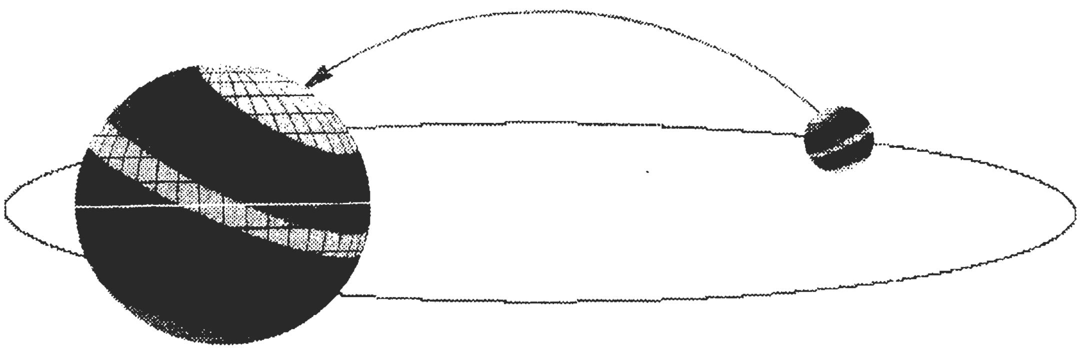

Figure 5: *Scheme of the opposition of a main star with its magnetic companion in a close binary system. The magnetic dipole is located outside the main star on the orbit and has no rigid connection.*

is influenced by the magnetic field of a companion, as shown in Fig. 7. The magnetic field of the ν Cep, detected by Scholz (1980, 1981), which could rarely preserve its magnetic field for physical conditions of a supergiant, is a candidate for such an indirect magnetic influence (Gerth et al. 2003).

All magnetic field configurations can be represented with arrangements of numerous magnetic dipoles (Gerth & Glagolevskij 2002).

13. Conclusions

The magnetic dipoles are regarded as the original sources of magnetic fields. However, this model can only describe the magnetic field structure without any distortion by the inhomogeneous distribution of chemical elements over the star's surface. The complexity of the structure of stellar magnetic fields leads to a very complicate entanglement of a lot of influencing magnitudes, conditions and procedures, which makes the inverse derivation of the map out of the observational measuring data extraordinarily difficult. The straightforward calculation is a valuable strategy to see already in advance what there is possible and how the map, the phase curve or the profile have to look. The method of modelling the magnetic field by the Magnetic Charge Distribution (MCD) offers a crucial advantage over the traditional calculation using spherical harmonics. The potential field is derived from its sources and calculated for the entire surrounding space including any plane like the surface of a sphere, whereas the calculation by an expansion of spherical harmonics with central dipoles, quadrupoles etc. is limited to the surface of the sphere and lacks of the physical meaning of the coefficients. The sources of the potential are the generating magnitudes as eigenvalues, from which a vector field like the magnetic field is derived. Therefore, the magnetic charges on point-like potential sources with its spatial coordinates are used as parameters. The mapping of a magnetic star can be carried out on the basis of mathematical treatment of a model of a star with few parameters in a straightforward calculation. A suited program is used as a tool, which enables the fitting of the calculated phase curves to the observational data and reduces the modeling of the magnetic field structure to the field sources.

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