Vertical structure of accretion discs in LMXB

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Abstract A low-mass X-ray binary (LMXB) is a binary system where one of the components is either a neutron star or a black hole. The other component (donor, usually a main sequence star) fills its Roche lobe and therefore looses mass to the compact object. We calculate the vertical structure of an accretion disc formed around the compact object. We develop a flexible numerical code, which allows us to change the equation of state, opacity law, and chemical composition. Our calculations can improve the accuracy of the modeling of LMXB outbursts.

Keywords: Accretion, Accretion Discs, Low-Mass X-Ray Binaries, LMXB, X-Ray, X-Ray Bursts, Vertical Structure of Accretion Discs

1. Introduction

Flares of LMXBs are among the brightest transient sources in the X-ray sky. To model X-ray bursts in LMXBs, one has to solve the equation of non-stationary disc accretion (see, for example, [1]). To solve this equation, a relation between physical parameters of the disc, for example, the energy flux and surface density, should be provided. Such a relation can be obtained by solving equations of vertical structure and finding dependencies of the pressure, surface density, energy flux, and temperature on the vertical coordinate.

Considering different physical conditions and processes in discs, we can construct so-called S-curves [2], namely, the dependencies of the radiation flux on the surface density, and study an instability driving outbursts of LMXBs.

Up to now, the structure of accretion discs was calculated by numerical methods in many works. For example, the vertical and radial structure of an accretion disc was calculated in [3].

In [4], the vertical structure was calculated, introducing Π-parameters (see below) and using analytical formulas for the opacity. We offer a more advanced and more flexible code, which uses modern opacities and is intended to be publicity open.

2 Model of the vertical structure

The equations of the accretion discs vertical structure are (see, e. g., [3], [4], and [5]):

$$\frac{1}{\rho}\frac{dP}{dz} = -\omega_K^2 z \ , \tag{1}$$

$$\frac{d\Sigma}{dz} = -2\rho \quad , \tag{2}$$

$$\frac{dT}{dz} = -\frac{3\varkappa_R \rho}{4ac} \frac{Q}{T^3} \ , \tag{3}$$

$$\frac{dQ}{dz} = \frac{3}{2} \omega_K \omega_{r\varphi} \quad , \tag{4}$$

$$z \in [0, z_0]$$

Here, Eq. (1) is the hydrostatic equilibrium equation with the pressure P, density ρ, Keplerian angular velocity $\omega_K^2 = GM/r^3$; Eq. (2) is, in fact, a definition of the surface density Σ as a function of the vertical coordinate z; Eq. (3) gives the temperature gradient in the case of radiative energy transfer in the Eddington approximation, where $\varkappa_R$ is the Rosseland mean opacity, Q is the radiation flux, a is the radiation constant; Eq. (4) is an equation of viscous heating, where $w_{r\varphi}$ is a component of the viscous stress tensor. The vertical coordinate z changes from zero (the plane of symmetry) to z_0, the semithickness of disc.

This system should be supplemented by an equation of state and law of viscosity:

$$\rho = f\,(P, T\,),\ w_{r\varphi} = \alpha P.$$

We use the Shakura-Sunyaev α-prescription for viscosity [6], where α is the turbulence parameter.

The code in Python 3 was developed to solve system (1–4) numerically. Input parameters are: the mass of the central body M, radius r, radiation flux F (or effective temperature T_{eff}) at this radius, type of opacity, turbulent parameter α.

The opacity coefficient $\varkappa_R$ is determined in the code by analytical formulas or by tabular values. In particular, we use MESA [7] for interpolating tabular opacities (opacity tables from [8], [9], and [10]). An example of opacity coefficient in the disc at one location is shown in Fig.1 (the right panel). At a fixed radius of the disc the opacity coefficient changes with temperature and density.

Boundary conditions for equations (1-4) are:

$$\Sigma(z_0) = 0,$$

$$Q(z_0) = Q_0 = \sigma_{SB} T_{eff}^4$$

$$T(z_0) = T_{eff}$$

$$P(z_0) = P_0$$

The program integrates the equations of vertical structure and performs an optimization task to find the values of free parameter z_0, for which the condition at the symmetry plane of the disc is fulfilled:

$$Q(0) = 0.$$

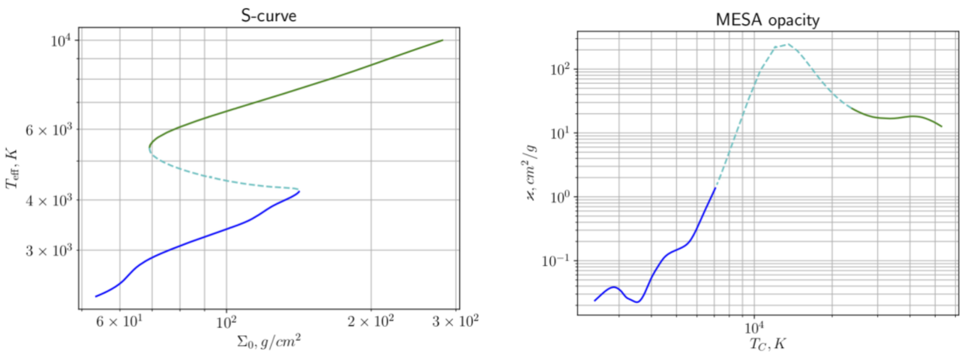

Fig1. *The left panel shows the S-curve for r = 10^{11} cm, α = 0.5, and M = 6M⊙. The right panel shows an example of opacity provided by MESA, calculated for the same range of parameters. Same colors on both panels show the same parameters. Region of thermal instability is indicated by the dashed line.*

3 Testing the code

The vertical structure (Eqs. 1–4) with the Kramers formula for opacity has been previously calculated in [4], where the dimensionless Π-parameters were introduced:

$$\Pi_1 = \frac{\omega_{\rm K}^2 z_0^2 \mu}{\mathcal{R} T_c}\ , \quad \Pi_2 = \frac{\Sigma_0}{2 z_0 \rho_c}\ ,$$

$$\Pi_3 = \frac{3}{4}\frac{\alpha \omega_K \mathcal{R} T_c \Sigma_0}{Q_0 \mu}\ , \quad \Pi_4 = \frac{3}{32}\left(\frac{T_{eff}}{T_c}\right)^4 \Sigma_0 \varkappa_c\ .$$

Here T_c, ρ_c, and $\varkappa_c$ are the temperature, density, and opacity coefficient at the symmetry plane of disc. In our code we obtain the values of Π that are in good agreement with the values calculated in [4] (see Table 1).

Table 1: *Comparison of Π at r = 10^{11} cm, α = 0.5, M = 6M⊙, T_{eff} = 2300K, ΔΠ — difference between the values from [4] and those calculated in our code.*

	Our code	ΔΠ
Π_1	5.7928	0.0028
Π_2	0.5373	0.0002
Π_3	1.1298	0.0002
Π_4	0.3994	0.0016

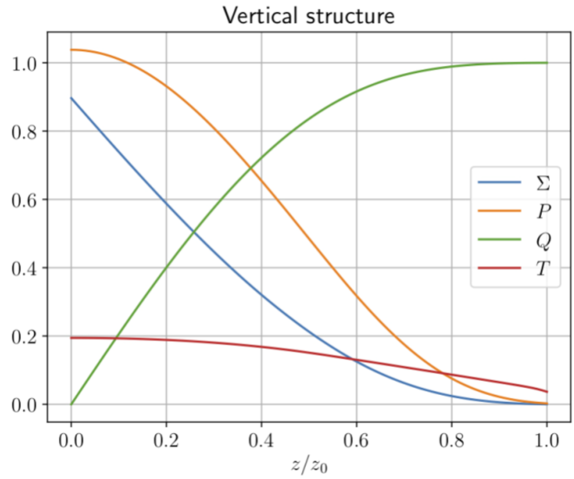

Fig2. *Calculated vertical structure for $r = 10^{11}$ cm, $\alpha = 0.5$, $M = 6M_\odot$, $T_{eff} = 10_4$K. All quantities are normalized to their characteristic values (see Eqs. (5)).*

4 Results

We show an example of the vertical structure calculated by our code in Fig. 2. This structure is obtained for $r = 10^{11}$ cm, $\alpha = 0.5$, $M = 6M_\odot$, and $T_{\rm eff} = 10^4$K. Opacities provided by MESA are used. All quantities are normalized to their characteristic values:

$$Q' = Q_0, \quad \Sigma' = \frac{28Q_0}{2\alpha z_0^2 \omega_K^3}, \quad P' = \frac{4Q_0}{3\alpha z_0 \omega_K}, \quad T' = \frac{\mu}{\Re}\omega_K^2 z_0^2 \tag{5}$$

We obtain the half-thickness of the disc $z_0 = 5.1 \cdot 10^9$ cm ($z_0/r = 0.069$).

The S-curve calculated for the same parameters of the disc if shown in the left panel of Fig1.

5. Summary

We are developing a modern flexible code to calculate the vertical structure of accretion discs for a wide range of parameters. Much effort is put into the code ability to easily change the opacity, chemical composition, and equation of state. The code is intended to be publicity available in the future.

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