Astrophysics and Space Science **197:** 179-212, 1992. © 1992 *Kluwer Academic Publishers. Printed in Belgium.*

THE PROPERTIES OF THE STRONG STATIC FIELD OF A COLLAPSAR IN GRAVIDYNAMICS

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(Received 2 December, 1991)

Abstract. In a totally nonmetric model of the gravitational interaction theory (gravidynamics) a compact object strong field (a collapsar) - an analogue to the black hole in General Relativity (GR) - is investigated. In the case of extremely strong (for gravidynamics) collapsar field a region filled by matter (a bag) must have a radius equal to $r_* = GM/c^2 </\approx 10$ km at the total collapsar mass $M </\approx 7M_\odot$. Only half of the total collapsar mass is contained in the bag, the other one of its total energy (Mc^2) is distributed in the form of a 'coat' in space around the bag, i.e., in the form of a continuous medium (as a relativistic 'gas') of virtual gravitons. The object must have a surface (the bag surface) with absolutely definite physical properties. The potential of such a surface is finite $(\psi_* = -c^2/2)$ and mass a particle in a bound state on the bag surface is two times less than mass of the same particle in a totally free state. The bag surface can undergo periodic oscillations (pulsations) with the period $GM/c^3 \approx 3 \cdot 10^{-5}$ s. An energy density inside the bag with the extremely strong gravitational field or with an extremely dense 'coat' (shrouding the bag) is determined by gravitation theory constants only and depends on the distance to the bag center r in the following way: $\varepsilon(r) = (c^4/8\pi G)r^{-2}$. In this case the bag matter is most probably in the state of quark-gluon plasma.

1. Introduction

This paper continues the study of collapsar physical properties in gravidynamics (GD), started in papers (Sokolov, 1992a, b; hereafter referred to as [P1] and [P2]). My purpose here is to elucidate such properties of the collapsar which must lead ultimately to absolutely definite experimental tests allowing us distinguishing the collapsar in GD from the collapsar ('black hole') in GR. Like previous papers, here I emphasize the connection of such objects problem in GD with the problem of localization and the sign of gravitational field energy.

In [P2] the following was used as an initial idea of the collapsar from the point of view of the dynamic field theories: The GD collapsar is a bound, spherically-symmetric object, a 'bag' filled by particles and fields together with the gravitational field surrounding this bag. Particles, bound in such a bag, with nonzero (may be) rest masses, move in such a way that both the bag itself and the field around it look like some stationary state in which the continuous exchange between the bag and the surrounding field occurs. The processes of the gravitational self-action play an essential and even determining role near the surface of the bag with dimension $\geq GM/c^2$. Here the quantity M determines first of all the total energy (Mc^2) of the whole configuration ('the field + the bag = the collapsar'), i.e., the collapsar total mass. At a given and constant value Mc^2 and with a constant energy in the bag, such an exchange must lead to settled/steady-state (definite and constant) values of energy-momentum-tensions for the static field in vacuum around.

For reasons mentioned later over and over again, we shall deal only with macroscopic (non-quantum) objects of the stellar mass order or greater. For such bags the stationary exchange, the bag ↔ the field, means the existence in vacuum of the static and spherically-symmetric gravitational field (as for the field of a point particle with the mass M), corresponding to a static (more exactly, stationary) and a spherically-symmetric matter distribution inside the bag.

Unlike GR, in GD there exists a possibility (in principle) for transition into such a stationary state with the spherically-symmetric and compact bag with the dimension of only several kilometers for stellar mass collapsars. In GD such a transition − a spherically-symmetric collapse − is accompanied by an energy loss of the collapsing body in the form of scalar radiation or scalar gravitons. With all this, a changing of the energy-momentum tensor trace of the whole collapsing system (the 4-scalar of the whole configuration) is the source of scalar waves.

In this connection, one can pick out and then sharply distinguish between two problems concerning collapse of a body:

The first one is the problem of the collapse process per se. I.e., this is the problem of the description of a non-stationary process (explosion) accompanied by an essential loss of initial total energy of a collapsing system in the form of gravitational waves. In particular, in whole such a problem will require also the allowing for effects of the braking of falling matter by radiation arising at the collapse.

The second problem may be reduced as a matter of fact to the justification of the very possibility of a steady (stationary) state existence, when one can neglect the radiation from the system as much as at least its influence on the value of total energy (Mc^2) of the collapsar become insignificant.

In this paper and in previous ones [P1, P2] the question is already basically on a result of the collapse - on the collapsar and its properties. In particular, we want to prove here that the collapsar, and, more correctly, the bag, may have also a solid surface. More exactly, it means that the outer border of the bag may, for some reason or other, be in the stable equilibrium state; i.e., it (the border) is static, for example, for the bag radius $R = r_* \equiv GM/c^2$. (That is impossible, in principle, in GR at $r = 2r_*$ already.)

Suppose that the collapsar with such a bag radius $R = r_*$ really exists, i.e., it is stable. Then this object in GD is defined first of all by its constant 4-scalar - the trace of energy-momentum tensor (EMT) of the whole configuration (because everything had collapsed and stabilized already). Ultimately, it means (as it was mentioned above) that the collapsar has the mass *M*. I am going to explain what I mean more exactly below.

The point is that (*firstly*) the field of a 'point' with the mass M at $r >> r_*$ is spherically symmetric and by force of correspondence principle this field is almost Newtonian one, i.e., it is determined by the same *M*. As a matter of fact (as [P1] has shown), Newtonian field of a massive 'point' consists by half of the field energy density θ_{00} in vacuum and it involves the repulsion scalar field with the spherically-symmetric potential

$$\psi(r) = -\frac{f}{8\pi a}\frac{M}{r} \ ,$$

determined by the quantity M also. (The notation is anywhere the same as in [PI, P2].) The scalar $\psi(r)$ is a solution of Poisson's equation for the massive 'point' at the center (at the origin of coordinate):

$$\Delta\psi = \frac{f}{2ac^2}T \ , \qquad (1)$$

where one must take the quantity $T = Mc^2\delta(r)$ (the trace of EMT of the whole configuration) as a source. The correspondence principle and the spherical symmetry of the field point 'acted' here.

But on the other hand (and *secondly*) the spherically-symmetrical repulsion potential $\psi(r)$ defined by the quantity *M*, will determine as before the scalar component of the collapsar field up to the bag surface of radius $R = r_*$. The field of itself does not contribute in the source – in the right-hand side of spherically-symmetric equation (1) – by force of the condition $\theta^{ik}\eta_{ik} = 0$ for the field EMT θ^{ik}, i.e., of the massless gravitational field condition. That is why at least out of the bag, in vacuum, the trace of the collapsar EMT will be zero as before. And inside the bag, by force of Poisson's equation feature, T may be a function only of r. Thus, for the function $T(r)$ we have $T(r) = 0$ at $r > R = r_*$ and $T(r) \neq 0$ at $r \leq R = r_*$ (so, δ-function became 'more definite'). Here the spherical symmetry and lack of the field mass (the zero-mass field condition) acted.

All in all, one can say that correspondence principle (the spherical symmetry of massive 'point' field and the massless gravitational field condition) allow us presenting a 'half' (at least) of static vacuum field of the collapsar as the scalar source T of two forms

$$T = Mc^2\,\delta(r) \tag{2a}$$

and

$$T = T(r), \tag{2b}$$

where in the first case (2a) one can take all r greater than r_*, and in case (2b) $T(r) \neq 0$ at $r \leq r_*$, i.e., inside the bag it is already a nonzero (positively defined) spherically-symmetric function up to $r = r_*$.

Accordingly, (2a) and (2b) mean that the integration along volume for T gives always the same result equal to Mc^2 for any surfaces in vacuum surrounding the bag completely.

Or, in other words, one can say that if an object with the bag of radius of $R = r_*$ is stable, then *at any distance* from the bag in vacuum we deal with the object (collapsar) having a *definite* rest mass M (and total energy Mc^2) at the usual Newtonian field of the point with mass M at infinity, i.e., at $r >> GM/c^2$.

I emphasize once more that here the question is basically on collapsar properties in vacuum, more exactly out of the bag - the sphere filled by matter. All the time I say about the vacuum potential, vacuum gravitational field, excluding the bag itself. Certainly, it simplifies considerably (and makes deficient in some degree) the collapsar problem, reducing it to the revealing of only gravitational properties of these objects. However, allowing for inevitability of some simplifications, one must say that the study of gravitational properties is first of all the study of a matter behavior in the collapsar field. In particular, one can speak about the matter behavior on the bag surface that is determined by gravitational field of the collapsar near this surface.

Though the most reliable conclusions of the paper concern the collapsar properties in vacuum, including the bag surface, one can (and must) go on, trying to understand how the bag itself is constructed from the point of view of GD. (I repeat again: we exclude here the quantum size of the bag.) Out of the sphere of radius $R = r_*$ the gravitational field remains the same, passing, in particular, to the spherical Newtonian field of point mass M at $r >> GM/c^2$. The matter (substance) distribution inside the bag must have spherical symmetry.

Elsewhere further (in the GD) I shall endeavour, in the style of all modern theories of dynamic field, to adhere to the idea that an 'elementary' point GD object (at $r >> GM/c^2$) is the collapsar indeed (at r >/~ GM/c^2) and that it consists itself of interacting (another 'fundamental') point objects. I.e., we shall always consider, at least for macroscopic collapsars, that inside the sphere of radius $R = r_*$ there are bound particles of rest masses $m^*_a \neq 0$ interacting by means of some *massless* fields. In accordance with that one can say that inside the bag there is matter with the equation of state *not harder* than $p = \varepsilon/3$ (Landau and Lifshitz, 1973). In other words (prior to possible violation of this requirement by quantum corrections generally speaking) the EMT trace of such an interacting particles system must correspond to the trace of the bound particles only. I.e., the spherically-symmetrical function $T(r)$ in (2b) inside the bag (at $r \leq R = r_*$) can always be presented as

$$T(r) = \mu_* c^2 \sqrt{1 - v^2 / c^2} \quad , \tag{3}$$

where $\mu^* = \sum m^*_a \delta(r - r_a)$ and v^2 are nonzero functions (depending on r) inside the bag only.

Certainly, in principle, at sufficiently small and even quantum dimensions of the bags the EMT trace of massless fields inside the bag becomes nonzero for sure because of the allowance for quantum corrections. But we emphasize everywhere that here the question is on the objects with sufficiently large (not less than several kilometers and more) dimension of the region filled by matter, where the corrections can still be terrifically small. Anyway, we shall endeavour to use condition (3) for the EMT trace of the collapsar until obvious contradictions arise in physical properties of the bag and the field around. (In such a situation the allowing for quantum effects inside the bag becomes inevitable.)

Eventually, in the following sections we shall answer to questions arise already in [P1]: What is the radius of the bag (in kilometers) for the last stable state with (still) static field outside the bag? How much energy is contained in the bag and how much in surrounding field? What is the potential on the surface of such a bag equal to? Do the collapsar properties depend on the value of M in fact or not (like properties of black holes in GR). As we shall see later, all these questions can be reduced to one: has the GD collapsar a surface and what are its properties?

In our previous paper [P2], supposing that the energy-momentum-tensions of both components of gravitation - tensor and scalar ones - are equal to each other in every point out of the bag, we obtained for the collapsar field in vacuum the 4-potential ψ_{ik}, whose nonzero components of the form

$$\psi_{00} = \frac{GM/f}{r}\left(1 - \frac{1}{2}\frac{r_*}{r}\right) \ ,$$

$$\psi_{11} = \psi_{22} = \psi_{33} = \frac{GM/f}{r}\left(1 - \frac{1}{6}\frac{r_*}{r}\right), \tag{4}$$

where as before $r_* \equiv GM/c^2$.

In this paper I shall endeavour to answer the question about a spherically-symmetrical equilibrium configuration - the collapsar; i.e., to understand the static field properties in vacuum up to the sphere $r = r_*$, proceeding from the fact that it is this potential which determines these properties. For that I will have to investigate the motion of test particles in potential (4) at least out of the sphere $r = r_*$.

In other words, here we approach the situation near the gravitational radius r_* from the point of view of equations of motion of particles in a given field of the form (4). Eventually we must have a self-consistent and complete description of gravitational properties of the collapsar following, on one hand, equations of motion and, on the other hand, field equations, constituting the basic system of equations of the GD [P1].

2. Test Particle in a Given Field of Form (4), Particle Mass in GD

In this section, with a view to main question about the possibility in principle of stable equilibrium on the sphere $r = r_*$, we shall look how the particle mass changes when 'immersed' in field (4), stipulating the possible role of gravitational radiation.

For the total energy of particle in the given field (4), using the general correlation obtained (Baryshev and Sokolov, 1983) we have

$$\mathscr{E} = \frac{mc^2}{\sqrt{1 - \upsilon^2/c^2}}\left(1 - \frac{f}{c^2}\psi^{0k}u_k\sqrt{1 - \upsilon^2/c^2}\right) =$$

$$= \frac{mc^2}{\sqrt{1 - \upsilon^2/c^2}}\left[1 - \frac{r_*}{r}\left(1 - \frac{r_*}{2r}\right)\right] \tag{5}$$

(the notation is anywhere the same as in [P1]). It follows from (5) that at a given particle velocity υ its total energy $\mathscr{E}$ is minimum at the sphere of radius of $r = r_*$.

As an example one can consider the (parabolic) case of the particle falling with the zero initial velocity υ_0 at $r \to \infty$ and conserved total energy. In other words, we neglect effects of gravitational radiation. From (5) we have for the particle energy $\mathcal{E}\,|_{r\to\infty} = mc^2$ = const. Then for maximum velocity which the particle can achieve at such a falling to the centre of sphere $r = r_*$, we have

$$\frac{mc^2}{\sqrt{1-v^2/c^2}}\frac{1}{2} = mc^2 \ , \qquad v_{max} = \sqrt{3}\ c/2 \approx 0.86\ c \quad . \qquad (6)$$

at $r = r_*$ and at $v_0 = 0$ for $r = \infty$.

Of course, everywhere here the fulfillment of the condition $m << M$ is meant, for field (4) can be assumed given.

We can note obvious difference of result (6) from what has been well known for a long time in the analogous case in GR. In the same situation with a particle falling to the centre, in GR (in the absence of radiation) the particle velocity becomes equal to the velocity of light at $\to 2r_*$ (where M is the mass of black hole in GR).

The velocity $v_{max} = 0.86c$ (at $v_0|_{r\to\infty} = 0$) must be understood as some limit but never achieved velocity of a particle falling (parabolically) to the centre. Generally speaking, at motion with a large velocity in the given field (4) *with a sufficiently large gradient* of potential nearby the sphere $r = r_*$, in principle it must arise gravitational radiation, taking away a part of energy of the falling particle. The velocity, really attainable at $r = r_*$, must be less than (6).

Thus, let us remark here that the total energy of a particle, generally speaking, can decrease at its 'immersion' to field (4) down to the depth $r \approx r_*$ i.e.,

$$\mathcal{E}\,|_{r\to\infty} = mc^2 > \mathcal{E}\,|_{r\approx r*}$$

In particular, the energies of the same (?) particle, resting at first at infinity and having come to rest for some reasons (including the loss for gravitational radiation), 'stuck' to the sphere $R = r_*$, differ two times, as it follows from (5) as

$$\mathcal{E}\,|_{r\to\infty} = mc^2 \ , \qquad \mathcal{E}\,|_{r=r*} = ½\, mc^2 \ . \qquad (7)$$

Thus, before we reach bottom of the potential well in field (4) to the full stop, to the 'merging' with the bag at the sphere $r = R = r_*$, the particle must lose *a half* of its initial rest mass for gravitational radiation (and, possibly, for other forms of radiation). I.e., in GD the motion of test particles in field (4) occurs with changing rest mass of these particles, as was noticed already in [P1].

In the following it will be convenient to use an analogue of Newtonian potential in GD, corresponding to tensor potential (4):

$$\varphi_{GD} \equiv -f\psi_{00}(r) = -\frac{GM}{r}(1-\frac{1}{2}\frac{r_*}{r}) \quad . \tag{8}$$

The behavior of this function is drawn in Figure 1. At $r >> r = GM/c^2$ equation (8) passes to Newtonian formula, and on the sphere $r = r_*$ we have

$$\varphi_{GD}|_{r=r*} = -c^2/2 \;;$$

and one can say that there is no 'Newtonian' potential deeper than $-c^2/2$.

Total energy (5) of a particle in the given field (4) can now be rewritten by 'Newtonian' potential

$$\mathscr{E} = \frac{mc^2}{\sqrt{1-v^2/c^2}}(1+\frac{\varphi_{GD}}{c^2}) \quad , \tag{5'}$$

where it is seen that the quantity $m\varphi_{GD}$ is the 'potential' energy (the total energy minus mc^2) of a test particle of mass m, resting (v = 0) at a given distance r from the centre. So far as the main purpose here is the possibility of a static (v = 0) in field (4), this more accurate definition of the meaning of the quantities $m\varphi_{GD}$ and φ_{GD} is essential for the following.

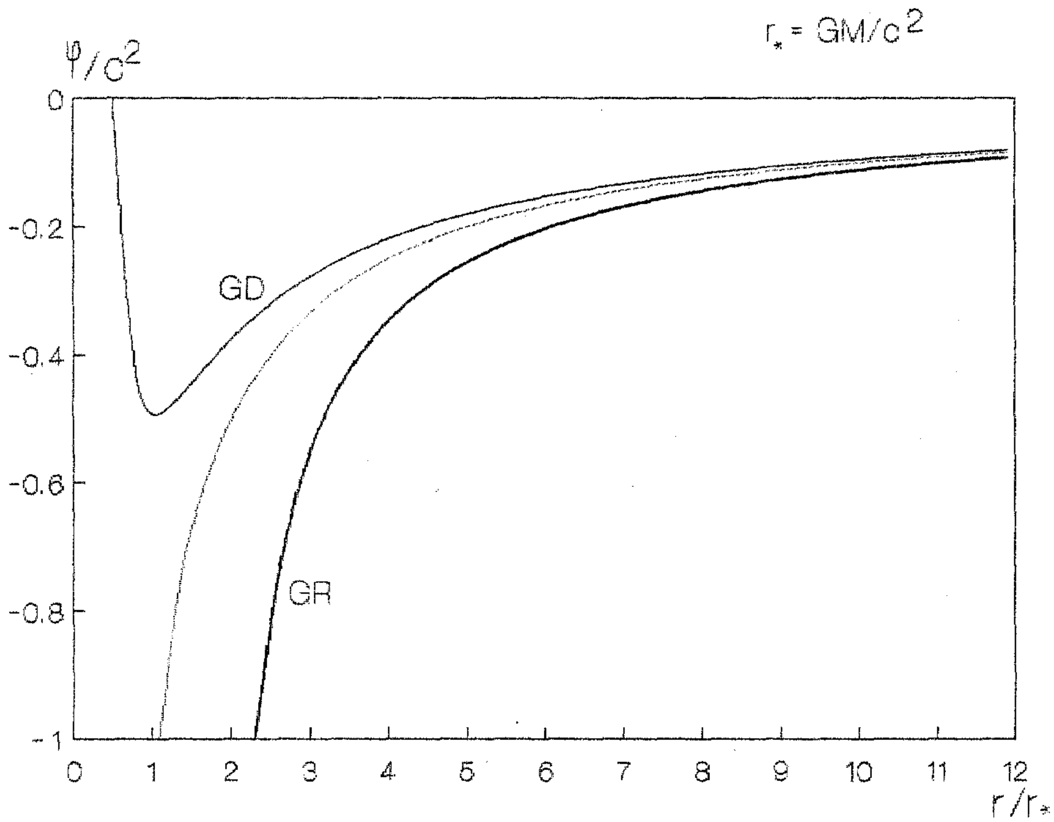


Fig. 1. The gravitational field potential *(φ)* in three theories (in units of c^2). The solid thin line is the collapsar potential in gravidynamics φ_{GD}, solid thick line is the 'potential' of GR, dotted line is the Newtonian potential.

The same energy can be written by the 0-component of the 4-vector $A^k \equiv \psi^{nk} u_n$ $(\varphi = A^0$, $\mathbf{A} = A^{\alpha})$ introduced in [P1] as

$$A^0 = \varphi = -\frac{\psi_{00}}{\sqrt{1-v^2/c^2}} , \quad \mathbf{A} = A^{\alpha} = \frac{1}{c}\frac{\psi_{11}\cdot \mathbf{v}}{\sqrt{1-v^2/c^2}} ,$$

$$\mathscr{E} = \frac{mc^2}{\sqrt{1-v^2/c^2}} + e\varphi \tag{9}$$

where $e \equiv fm$.

Now the difference between the φ_{GD} and φ-component of the 4-vector A^k is well seen (0-component of the A^k is an analogue of φ-component of the field in electrodynamics). The value of φ at $r = r_*$ depends, in particular, on the particle velocity, that is not very convenient. For example, if we assume that the limit value of velocity, which a particle can achieve falling from infinity to the centre with the zero initial velocity, does not exceed the value (6) $v_{max} = \sqrt{3}\, c/2$ then the maximum 'depth' for $A^0 = \varphi$-component would be equal to

$$f\varphi|_{r=r*} = -\,c^2 .$$

Formulae for $\mathscr{E}$, obtained above, differ only in notation. It is more essential to select a suitable and convenient formalism, allowing us comparing results of GD with conclusions of GR and Newtonian theory. In that case the φ-potential turns out to be inconvenient because of its dependence on the particle velocity, as was said above. In the following I shall endeavor to present the particle motion, analogously to classical mechanics, as a motion in potential (8) drawn in Figure 1.

In Figure 1 the classical Newtonian hyperbola ($-\,GM/r$) is drawn by the dotted line as a 'gauge' curve. It is well seen that sooner or later at the approach to the centre the potential φ_{GD} goes to the left from the classical hyperbola because of the influence of gravitational field energy-tensions continuously distributed in space around the bag (for more details see [P2]). The corresponding analogue of Newtonian potential in GR must go to the right from the 'gauge' hyperbola $-\,GM/r$, for its derivative would inevitably become the infinity in the point $r = 2GM/c^2$.

Generally, very essential is the fact (I shall imply it elsewhere below) that the quantity M at $r >> GM/c^2$ means the same in all three theories (GD, Newton, GR) that, of course, is a consequence of the correspondence principle. It is always the mass measured by usual astrophysical methods, or the mass in the usual dynamic sense, i.e., the mass not only as a gravitational 'charge'. The details, defining more exactly this notion of the collapsar mass, arise at the approach to the sphere $r = r_*$. Ultimately, papers [P1, P2] and this one are dedicated in any way to the clarification of the meaning of the quantity M.

In particular, when the static field of an object with $R = r_*$ is spoken here about, I always mean the existence of the classic limit for this field, i.e., the mass can always be understood in the usual classic sense as the mass of a matter point (see Introduction). If such a limit does not exist for some reason or other (for example, at $r \to 0$ and $T \to 0$, see [P1]), then, accordingly, one may not already speak about a static, or stationary at $r = r_*$, gravitational field. But eventually, then one may not already speak also about a point with the mass M. We shall return to this fact later.

As was marked before (see (7)), the motion of a test particle in field (4) occurs in such a way that, generally speaking, the particle mass in the equations of motion cannot be already assumed constant. And for the following it is very important to realise exactly the two moments.

On the one hand, at the 'merging' (or at full stop on the sphere $r = r_*$) of a test particle with a gravitating centre the energy lost for radiation must be large enough. It cannot be neglected if we consider, for example, the fall to the centre (and acceleration in field (4) near the sphere $r = r_*$) of the particle with an arbitrary velocity.

On the other hand, we can be interested in the possibility of a steady state, in particular for $r = r_*$, when obviously there is no radiation. Is such a state of rest $(v = 0)$ of the test particle on the sphere $r = r_*$ possible? We can also consider cases of stationary states of particles near $r = r_*$ with velocities and accelerations still small enough for it would be necessary to take into account the radiation. So far as gravitational radiation is still negligible at velocities $v^2/c^2 << 1$ and even at $(v^2/c^2)^2 << 1$, then in the overwhelming majority of practically important cases one may use the equations of motion in the field (4) with a given mass of test particle, i.e., with a definite rest mass (m_*) which does not change during the motion. It is also considered below in detail.

This last statement agrees with what was said in Introduction about the collapse and collapsar. Ultimately, the very process of the pass of a particle in a stationary (bound) state at $r = r_*$ (the fall to the bottom of the potential well in Figure 1) can be still considered not in the whole volume. Below we shall basically speak about possible steady stationary states both of a collapsar and test particle, when there is no radiation already, or when it is sufficiently small - analogously to quantum mechanics of atom when one studies the stationary states of an electron at first.

One must keep in mind that any object before reaching ('merging' with) the sphere $r = r_*$ changes somehow its structure so that its total rest mass would become two times less than the rest mass of the same object at infinity.

Thus, the mass in GD depends on distance at which it is measured. Such a result was met already in theory with the self-acting field of gluons in QCD for quarks strongly bound inside hadrons. Hereafter I shall endeavour to emphasize analogous properties of these two theories with massless and self-acting gauge quanta: GD with two types of real gravitons and QCD with 8 types of gluons.

3. The Force Acting on a Test Particle near the Sphere $r = GM/c^2$ and the Possibility of Equilibrium on this Sphere

Let us begin with the simplest case - statics: i.e., let us assume that a test particle rests already on the sphere surface of the radius $R = r_*$. What forces act on it in that case? It is to the point to remember here an analogous problem for the black hole in GR about the particle, 'lowered by the rope': Is the rest possible at $r \to 2GM/c^2$?

Using the vector $\mathbf{A} = -A_\alpha$ and $\varphi = A_0$ from (9), one can write down for the force acting on test particle in a given field, in the form known from electrodynamics:

$$\frac{d\mathbf{p}}{dt} = e(\mathbf{E} + [\frac{\mathbf{v}}{c} \cdot \mathbf{H}]) \ , \tag{10}$$

where as usual $\mathbf{E} = -\partial\mathbf{A}/\partial t - \nabla\varphi$, $\mathbf{H} = \mathrm{rot}\,\mathbf{A}$, and $e \equiv fm$.

The difference from electrodynamics is (and it is essential) that there is a dependence on the particle velocity in vectors **E** and **H** already. But as far as we assumed that a particle rests ('lowered by the rope' in field (4) - the case of statics), then at $\upsilon = 0$ the 'inconvenient' vectors **A** and **H** disappear from (10). (The vector **A** does not depend explicitly on time by force of the static character of potential (4) and, hence, $\partial\mathbf{A}/\partial t = 0$, i.e., *the field is given*). As a result we obtain at $\upsilon = 0$:

$$\varphi = -\psi_{00}(r), \quad \mathbf{E} = +\nabla\psi_{00}(r), \qquad \mathbf{A} = 0, \qquad \mathbf{H} = 0. \tag{11}$$

I distinguish here especially this case in connection with the question posed at the beginning on the very possibility of statics at r ~/= r_* . But, strictly speaking, case (11) corresponds to the fact that for the gravitational force in Equation (10) we neglected all the ratio υ/c power expansion terms, beginning with υ^2/c^2. As far as in GD (unlike electrodynamics - ED) there are no terms proportional to the first power of the ratio υ/c $(\partial\mathbf{A}/\partial t = 0)$, then the case in question of 'statics' (11) allows, generally speaking, motions with low velocity $\upsilon/c \neq 0$, if only the second power of this ratio would be still sufficiently small: $\upsilon^2/c^2 << 1$. For example, 'statics' (11) can describe cases of motion with sufficiently large velocities υ </~ 0.1c = 3×10^9 cm s^{-1} , but it depends on a concrete/specific problem conditions.

So, such an 'automatic' dying-out of force terms, proportional to υ/c is a feature of GD. In general, there is nothing surprising here. The matter is that effects, connected with gravitational radiation presence (when one cannot assume, in particular, that $\partial\mathbf{A}/\partial t = 0$, and the field is given) become important in GD only when allowing for terms, in Lagrange function for a test particle in the given gravitational field, of the order (even) higher than $(\upsilon^2/c^2)^2$. Exactly because one can use 'static' case (11) for the given field (4) at $\upsilon^2/c^2 << 1$ always and not take into account 'a magnetic term' in equations of motion (10) and also corresponding addition in the vector $\mathbf{E}$.

Thus, we can write for the force acting on a test particle in the field with potential (4), discarding terms of the order of υ^2/c^2 (and higher) in (10):

$$\mathbf{F} = e\frac{\mathrm{d}\psi_{00}(r)}{\mathrm{d}r}\mathbf{r}_0 = -m\frac{\mathrm{d}\varphi_{GD}}{\mathrm{d}r}\mathbf{r}_0 = -\frac{GmM}{r^2}(1-\frac{GM/c^2}{r})\mathbf{r}_0\,. \tag{12}$$

As usual, $\mathbf{r}_0$ is directed from the centre. In this formula there is no dependence on the particle velocity but it is necessary to keep in mind that it is fulfilled till one can

assume that $\upsilon^2/c^2 << 1$. Only in that case the influence of additions to the force (and in particular of 'magnetic' addition) is still sufficiently small. For example, if small additions to the force of the order of $\upsilon^2/c^2 = 0.07$ can be still assumed negligible in the conditions of a given problem, then it means that formula (12) can be used at the study of motions in field (4) with velocities $\upsilon \approx x$ 0.26c = 80000 km s^{-1}.

Thus, with corresponding restrictions, formula (12) for the force can be applicable in many practical important cases of the particle motion in a given spherically-symmetrical field of form (4). It is particularly essential if one means the cases of motion of macroscopic objects in gravitational field with velocities though big, but still sufficiently far from ultrarelativistic limit, when $\upsilon \to c$.

Subsequent sections will show that in (12) the restriction must exist of r on the side of small $r < r_*$, connected with the impossibility of application of the very 4-potential (4) at too small r. But from (12) it is seen that at least at $r > 0$ the force acting on the test particle is always finite and $r = r_* = GM/c^2$ *it simply becomes equal to zero*. Thus, the field described by vacuum potential (4) allows, in principle, the possibility of stable equilibrium of the particle on the sphere $R = r_*$ as it is seen from formula (12) and Figure 1.

At a further decrease of the distance to the centre $(r < r_*)$, for example, by the compression of the central object (the bag) the repulsion must arise and, consequently, the point of minimum energy of particles (5) on the sphere $r = r_*$ is simultaneously the position of stable equilibrium for such particles. In other words, the vacuum potential (4) does allow the existence of the surface of the collapsar even at the bag radius $R = r_*$, i.e., the bag boundary can be at $r = r_*$ in the state of stable equilibrium.

It differs radically from what GR gives in the case. In GR (see Figure 1) the forces at distances $r \approx 2GM/c^2$ (i.e., even earlier and at the same M) will tend to infinity in the frame of reference in question - no equilibrium is possible in principle. It is necessary to say that GD, consistently allowing for the energy (or the self-action) of gravitational field, leads to rather unusual situation at $r = GM/c^2$. The feature of GD is the fact that though we have an object with a strong gravitational field over its surface ($\theta^{00} \approx Mc^2/r^3$ and with a strong attraction over the sphere with $R = r_*$), but the surface itself of such an object (the bag) at $r = r_*$ is in the region of total equilibrium of gravitational forces, i.e., at $r = r_*$ upper layers of such an object do not press at all on lower layers. It is absolutely different from what we have got used to in Newtonian gravitation, and from what GR gives, asserting that the weight of upper layers of contracting object is always only increasing at the decrease of the dimension of the object with a given M.

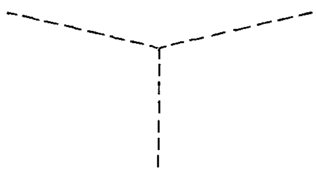
Ultimately, all investigated modifications of the equation of state of compact objects matter, I mean so-called 'realistic' equations of state for the interiors of neutron stars and, possibly, quark stars, must guarantee stable hydrostatically equilibrium configurations at condition of strong pressure of upper layers on lower ones. In GD the situation may turn out to be absolutely opposite for the object having attained the sphere $r = r_*$. In that case the equation of state of the matter inside the bag must correspond to the total absence of the pressure of upper layers on the lower ones of such matter. In other words, on the surface of the bag with radius $R = GM/c^2$ the gravitation is already 'turned out'.

Thus, if the description of the collapsar field is assumed by means of vacuum potential (4) for the particle, 'slipped in' the region of the sphere $r = r_*$ (of course, with the loss for radiation), then near the equilibrium state the gravitation as if disappears. There is no attraction on the sphere $r = r_*$, though θ^{00} is large. In that case the situation begins to remind the state of asymptotic freedom for bound quarks in a hadron in QCD.

The analogy with QCD may be intensified, if to mean the decrease of the rest mass of the test particle (see Section 2), 'immersed' in field (4). In GD also one can consider that the masses of particles bound in the bag are less than the masses of the 'same' particles measured at distances between them much more than r_*.

Thus, in GD in the case when the distance between bound particles becomes one of the order of GM/c^2, their masses decrease and the forces of gravitation (and we speak for the present only about the gravitation) acting on such particles, can tend to zero.

In general, such a picture of bound particles behaviour could be expected at once for the theoretical scheme, which allows consistently for the self-action of gravitons. In particular, in QCD one of the consequences of essential nonlinearity of the theory, showing itself in the self-action of gluons, is the property of asymptotic freedom, i.e., of the decrease of the force of quarks interaction at their rapprochement till very small distances. Thus the processes of the type of



played its role.

However, the mention of QCD is here only a more or less suitable analogy. The equilibrium of the scalar force of repulsion $\mathbf{F}_{(0)}$ and the tensor force of attraction $\mathbf{F}_{(2)}$ is the device which 'turns off the gravitation on the sphere $r = r_*$ in GD. For all this, the energy densities $\theta^{00}{}_{(0)}$ and $\theta^{00}{}_{(2)}$ of both components of gravitation, equal to each other, achieve the maximum on the surface with $R = r_*$:

$$\theta^{00} = \theta^{00}_{(0)} + \theta^{00}_{(2)} = \frac{1}{16\pi}\frac{GM^2}{r^4} + \frac{1}{16\pi}\frac{GM^2}{r^4} = \frac{1}{8\pi}\frac{c^8}{G^3M^2} \ .$$

As was done already in [P1], one can present force (12) as an algebraic sum of forces acting on test particle in field (4), i.e., it will be the sum of forces: $\mathbf{F}_{(2)}$ - the force arising because of the presence of the tensor component of gravitation in (4), and $\mathbf{F}_{(0)}$- the force arising because of purely scalar component of field (4). For the 'tensor' force we have

$$\mathbf{F}_{(2)} = -\frac{3}{2}\frac{GmM}{r^2}(1-\frac{2}{3}\frac{Gm/c^2}{r})\mathbf{r}_0 \ . \qquad (13)$$

And for the purely scalar component of field (4) the repulsion stays as before because of the linearity of the scalar field at the condition $\theta^m{}_m \equiv 0$, which guarantees the existence of the classic limit of (4) to be

$$\mathbf{F}_{(0)} = \frac{1}{2}\frac{GmM}{r^2}\mathbf{r}_0 \ .$$

Now it is seen that at $r = r_*$ the total force for potential (4) $\mathbf{F} = \mathbf{F}_{(0)} + \mathbf{F}_{(2)}$ becomes zero, i.e., the gravitation (tensor component (4)) is compensated on the sphere $r = GM/c^2$ by the scalar 'anti-gravitation'.

The opportunity of the usage of formulae (4) for the vacuum potential, when the bag radius becomes less and essentially less than $r = r_*$ will be studied especially in the following sections. But to anticipate the end of the paper I say that the vacuum potential (4) and formula (12) are applicable till one can speak about their classic limit. Correspondingly, for the potential and the force of attraction around 'the point' with the rest mass equal to M in the space with $r >> GM/c^2$ we must have formulae (29) and (35) from [PI], i.e., Newton's law. Ultimately, the bottom limitation of r follows the fact that the total mass of the configuration (the collapsar) cannot be the field one only, when the classic limit of (4) is absent.

Though here the question is always on the classical theoretic scheme for the gravitational field of the collapsar with vacuum potential (4), but so far as the dynamic treatment of gravitation constitutes the base of such a scheme, I shall have (as in [P2]) to touch more or less profoundly upon the very nature of gravitational interaction. The nature of dynamic (gauge) fields must be thought about with the usage of quantum notions, quantum elementary processes with the participation of photons, gluons, weak bosons and, lastly, gravitons (of tensor and scalar form).

If we are not too dogmatic about GR, one could make up his mind to express the following. It seems to me, that purely classic description of gravitation phenomenon in GR by means of curves space-time, now (from the point of view of the theory of dynamic fields) can be assumed good but only phenomenologically.

Here we can recall that up to recently for the description of weak interactions the scheme of 4-fermions interaction served well and for a long time. Time came, and one had to think about the real theory. Eventually, a new scheme was created, unifying two interactions in the theory by Weinberg-Salam. An attempt to understand more profoundly the nature of weak interaction led to the introduction of weak bosons into the theory - the theory became more general and more exact as a result.

By means of this (lyric) digression I want to emphasize once more that attempts to understand the gravitation leads inevitably to the usage of the experience of classic GR on one hand, but on the other hand one cannot do so without a resort to all quantum-field theories.

How close we approach the understanding of the nature of the gravitational field in the proposed scheme here (and in [P1, P2]) with one more, massless scalar graviton?

I should like to think that like other field theories and in general as it is in high-energy physics, the answer to this question will be given in corresponding experiments (observations). Anyway, the appearance of fundamental scalar elementary particles in the theories with spontaneous violation of gauge symmetries is considered now (almost) inevitable.

As to the massless scalar graviton, which naturally appears in GD and which is an essential component of gravitational interaction in the scheme, proposed here, then, as it was shown in [P1], such a scalar is naturally connected with the total mass of the whole configuration - the collapsar by means of a source

$$T = Mc^2\delta(\mathbf{r}) = \mu_* c^2\sqrt{1-v^2/c^2} \ .$$

In this connection it is to the point to recall the source of Higgs's scalars.

In the conclusion of this section I remark that the notion of virtual gravitons (scalar and tensor ones, see [P2]), which led eventually to vacuum potential (4), arises naturally in that idea environment, which modern physics of high energies created: QED, QCD, etc. It are these ideas, constituting the base of dynamic interpretation of fields, which lead to the presentation of gravity field as a totality of two components - the scalar and the tensor ones. It is from here something arises which looks like an asymptotic freedom of QCD at $r \approx r_*$.

4. The Scale of Forces, a Maximum Acceleration of Gravity Force Near the Collapsar Surface and the Sphere of Maximum Instability for a Given Mass *M*

It is difficult (though it may be appropriate) to keep from referring once more to QCD, but an opinion is expressed already rather often, that apparently the same nonlinearity of the theory leads to the fact, that at distance more or of the order of a neutron dimension the force of attraction between quarks becomes so large that it does not allow to quarks to be free. As is seen from Figure 1, the field with vacuum potential (4) in nonlinear GD gives for φ_{GD} something like the confinement of QCD. But here the difference between the potentials φ_{GD} for a bound particle $(r \approx r_*)$ and for a free particle of GD in finite ($\leq c^2/2$), and the attraction force, acting on the test particle, 'immersed' in field (4), cannot exceed a certain maximum value.

The largest force of attraction acts on particles situated on the sphere $r = 3/2\ r_*$ - in the point of maximum gradient of the potential φ_{GD} in Figure 1. This force is equal to

$$\mathbf{F}_{\max} = -F_G \frac{m}{M} \cdot \mathbf{r}_0 \quad \text{for} \quad r = r_{max} = 1.5GM/c^2 \quad , \tag{14}$$

where
$$F_G \equiv \frac{4}{27} \frac{c^4}{G} \quad ; \tag{*}$$

and F_G can be understood as a limit possible force of attraction acting between two points with equal masses, situated at the critical distance $l.5GM/c^2$ to each other.

The difference between the maximum force of attraction F_{max} for a given M and the limit possible force F_G is emphasized the best by reasoning following hereafter.

In any cases in question it is implied that vacuum field (4) is the given field. It means that the test particle of mass m moving in this field disturbs weakly the movement of the massive gravitating centre. That is why the gravitating object can be assumed to be at rest with the centre to be in the origin of the frame of reference - practically it is this fact which defined (see in [P1]) the frame of reference. If we shall assume that in that case the condition is fulfilled

$$m << M$$

(and it was always implied before), then the words 'the given field' or 'the given frame of reference' mean in fact (see (14)) that

$$\frac{m}{M} = \frac{F_{\max}}{F_G} \ll 1 \tag{15}$$

on the sphere $r = r_{max}$.

In these cases I mean always the given static field with vacuum potential (4), i.e., here we call test particles the particles for which even on the sphere $r = r_{max} = l.5GM/c^2$ the force of attraction is still sufficiently small in comparison with the limit possible force of attraction F_G.

Thus, the introduction of the limit possible force of attraction F_G makes absolutely definite such notion as the test particle (the passive attracted mass), the given field (the active attracting mass in the expression for $r_{max} = l.5GM/c^2$) and, ultimately, the question about the special status (or the distinguishing state) of inertial frames of reference in GD can be considered cleared up definitively.

So, in GD the scale for the force of attraction is introduced quite unambiguously, and the quantity $F_G \approx$w 1.79 x 10^{48} dynes plays the role of the fundamental force.

However, it is not worthwhile to hurry with the direct comparison of F_G with other forces existing in nature. Here it is very essential to keep in mind the radius of rapprochement $1.5GM/c^2$ between two gravitating objects.

So far as the force of attraction between two objects cannot exceed the value $F_{max} = F_G(m/M)$ and it depends on the ratio of masses of these bodies (with the attracting active mass to be in the denominator and the attracted passive mass to be in the numerator), then this force of attraction never is more than F_G. It is excluded simply because otherwise the attracting body and the attracted one will exchange their roles.

Of course, one should not forget that the condition $m/M << 1$ for the active and passive masses corresponds to the given static field (in a fixed frame of reference). The cases, when the ratio m/M becomes of the order of 1, mean as a matter of fact (in conditions, when the radius of rapproachement ~ r_{max}) the pass to situations with nonstatic gravitational field already. Such problems inevitably will demand the allowing for the role of the gravitational radiation. (And not only for the tensor one, as it was with the double pulsar PSR 1913 + 16 at rather large still dimension of the orbit.)

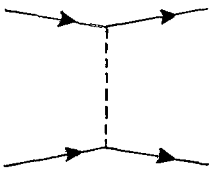
As it was emphasized many times before, the movement of particles at relative distance to each other of the order of Gm_a/c^1 will demand ultimately the consideration of objects with variable masses. At $r \sim GM/c^2$ the movement, for example, of two equal masses take place in the region of nonlinear GD, when their gravitational 'coats' essentially intersect each other.

I endeavour here, in this paper, to restrict the range of problems only to stationary ones, and in the limit - to static situations without radiation. But nevertheless I wish to remark that the limit possible force of attraction F_G is equal everywhere both for stellar masses and for masses of elementary particles.

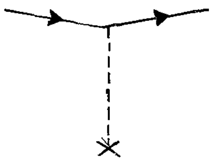
The other matter is that the rapprochement, for example, of two nucleons to the critical radius $1.5GM/c^2 \sim 10^{-52}$ cm would require 'to turn out' at first all other interactions existing in nature. In fact, the nucleon will cease to be a nucleon much earlier. Thus, the limitation arise of really achievable value of gravitational interaction F_{max}, connected ultimately with the existence of other interactions.

Before going further, I remark that the condition $F_{max}/F_G = m/M << 1$ is naturally connected also with the fact that we consider still not too small M, corresponding to nonquantum, macroscopic dimensions of bodies. Otherwise, reducing the mass unlimit-edly, we shall have to abandon the presentation of the gravitational field of the object in the centre as some continuous medium with the tensor θ_{ik}. Ultimately, one will have to consider an essentially quantum situation, in which the condition $\theta_{ik}\eta^{ik} = 0$ can be broken.

The essentially quantum situation implies the consideration of the processes of type of



with particles equal in rights instead of processes



(static potential).

For all that the movement of all particles-sources must be considered already within the bounds of quantum theory.

In our case of large (macroscopic) GM/c^2 the movement of test particles in macroscopic volumes stay also quite classical.

Thus, in what follows, the question is only on macroscopic gravitating objects and, accordingly, for M the values of the order of stellar masses and more are taken. Then for the maximum acceleration of free fall on the sphere $r = r_{\max} = 1.5GM/c^2$ (the sphere of maximum instability) in vacuum for the object with the mass M we have

$$g_{\max}(r = r_{\max}) = -\frac{4}{27}\frac{c^4}{G}\frac{1}{M} \quad . \tag{16}$$

In accordance with (11) it is simply the tension of the gravitational field on the surface of the sphere $r = r_{\max}$ and here I especially draw the reader's attention to the fact that the more the mass M of the object, the less the tension of the gravitational field, maximally possible at this value of M. Below some estimations will be adduced, having for the purpose to emphasize of this feature of vacuum potential (4).

But at first we estimate the Newtonian acceleration on the surface of the stable neutron star with $M = 1.4M_{\odot}$ and the radius $R_{NS} = 10$ km by usual formula $g_{NS} = -GM/R^2$. It is equal to $g_{NS} = 2 \times 10^{14}$ cm s^{-2}. The mean density of such an object is equal $\bar{\rho}_{NS} = 6.7 \times 10^{14}$g cm^{-3}.

It is necessary to say that the density of the matter of neutron stars is defined by the equation of state $P = P(\rho)$ *in* corresponding equations of hydrostatic equilibrium, allowing, in particular, for relativistic effects also, usually in the bounds of GR. For all this the macroscopic density of the matter achieves the values 10^{14}-10^{15} g cm^{-3} - nuclear and supernuclear densities.

All modern calculations (Shapiro and Teukolsky, 1983) show that at the acceleration on the surface of the order of $g_{NS} = 2 \times 10^{14}$cm s^{-2} one can choose such a law of changing of the pressure *P(r)* and of the density *ρ(r)* inside the star *(P = P(ρ))* which secures the needed gradient of pressure in case of hydrostatically-equilibrium neutron star. Here I especially choose some average parameters of the neutron star, which (for example) are often used at the interpretation of observational manifestations of such objects. The main fact is here that the object with the acceleration ~2×10^{14}cm s^{-2} on the surface can provide a quite stable formation.

In GD for the object with the mass $1.4M_{\odot}$ on the sphere $r = r_{\max} = 3.24$ km (still in vacuum) the maximum acceleration $g_{mzx} = 6.24 \times 10^{14}$ cm s^{-2}. From the point of view of GR the object is within its Schwarzschild radius (4.3 km) and it is impossible in principle to secure any equilibrium at any equation of state.

However, from the point of view of GD the acceleration g_{max} (6.24×10^{14} cm s^{-2}) is only the maximum acceleration for the mass $M = 1.4\ M_{\odot}$ and one can say about the macroscopic density of matter inside the bag of the dimension (the radius) of the order and *less* than R = 3.24 km. With the allowing for the field energy around the bag, such a density will be about 15 times more than ρ_{NS}.

But the question arises what kind of matter will be in the bag and what equation of state we can speak about in that case? What forces define it now?

Here I have to recall QCD again. According to QCD, at distances between the baryons less than 10^{-13} cm (it is characteristic radius of the nucleon) and, correspondingly, at densities exceeding the nuclear one ($p_{nucl} = 2 \div 2.8 \times 10^{14}$g cm^{-3}) a few times, the matter of the bag must undergo the phase transition in the state of quark-gluon plasma (QGP). As a result we deal with the degenerated Fermi-liquid, to the equation of state of which a lot of papers are dedicated now (see the literature in Chernavskaya and Chernavsky, 1988; Emel'yanov *et al.,* 1990). It can be assumed that in GD the neutron matter in the whole volume of the bag for the object with $M = 1.4\ M_{\odot}$ and $R \leq 3.24$ km must be already in the state of QGP. The further concretization of the bag properties from the point of view of GD deserves to be dealt in a special paper and the bag properties will be stated many times later. But now I am interested first of all in its gravitational manifestations in vacuum.

By use of (16), for the bag of the radius not exceeding 10 km but with the mass $M = 4.5\ M_0$, one can obtain the same acceleration (2×10^{14} cm s^{-2}) as on the surface of a 'usual' (stable) neutron star. According to GR it is already a black hole ($2GM/c^2 \approx 13.3$ km), i.e., in GD the hydrostatic stability of the object with $M = 4.5\ M_{\odot}$ and $R = 10$ km can be secured apparently by an approximately same equation of state as for a usual neutron star with the mass 1.4 M_Θ and the radius 10 km.

For not to worry for the present over parameters, indefinite in many respects, of the phase transition in the state of QGP (what will happen at densities ~ $5\rho_{nucl}$?), one can estimate the parameters of the 'superdense' neutron star taking one's cue from the mean density of usual neutron star with the mass 1.4M_Θ, the radius 10 km and $\rho_{NS} = 10^{14}$-10^{15} g cm^{-3}. If the matter of such an 'average' star undergoes a phase transition, it occurs somewhere in a small region near its centre.

Thus for the object with a mass M = 5.7 M_Θ and the radius $R = r_{\max}$ = 12.7 km the acceleration of free fall on the surface will be not greater than $g_{\max} = 1.6 \times 10^{14}$ cm s^{-2} with an average density of matter in the sphere of the radius R equal to the density of a usual neutron star. Here it has been already taken into account (see Section 6) that approximately a half of the total mass of an object is distributed around 'the bag' in the form of the energy of gravitational field (the energy of 'the gas' of virtual gravitons). One can think that here also the matter density in the bag changes approximately in the same range as for a usual ('average') neutron star. The phase of QGP is not developed yet, the basic part of the bag mass is distributed with the density 10^{14}—10^{13} g cm^{-3} and thus in the equation of state the same indefinitions remain which remain still in case of an 'average' neutron star also (Shapiro and Teukolsky, 1983).

For such a supermassive neutron star the neutron matter will constitute only 2.85 M_Θ (i.e., a little more than a half) of the whole object, the rest of mass will be in a purely gravitational 'phase' - the coat of virtual gravitons. Nevertheless (for example, in close binary systems) at $r >> r_{max}$ the total mass M = 5.7 M_Θ will be measured.

The question about properties of such objects and collapsars, having attained the limit small dimension (i.e., the sphere of total equilibrium) with the bag of the radius $R = r_* = GM/c^2$, is ahead. Here I would like to remark that in GD a stable compact object is possible, which looks like the black hole of GR by the measured mass M and the radius. But nevertheless it is a static object with a finite value of the gravity force on the surface and a quite acceptable (finite!) value of the matter density. The equation of state $P = P(\rho)$ can be almost the same, similar one, as in case of an 'average' neutron star.

If to increase more the mass in (16), then for the object with mass $M \sim 3 \times 10^6$ M_Θ and the radius ~ 10 R_Θ (R/c ~ 0.5 min) which in GR inevitably collapses in the black hole, the acceleration on the surface is equal to $g_{max} \approx 3 \times 10^8$ cm s^{-2}. It corresponds to the acceleration of the gravity force (GM/R^2) on the surface of a white dwarf. The mass density (~ M/R^3) of such an object is only < 5×10^3 g cm^{-3} - i.e., by three orders less than a density of white dwarfs (~ 10^6 g cm^{-3}). Certainly, such an object cannot be already called a supermassive white dwarf.

For a body with the mass 7 x $10^{10}M_{\odot}$ and the radius 1.6 x 10^{16}cm ~ 10^3AU *(R/c ~ 6* days) the acceleration on the surface is not greater than the gravity acceleration on the surface of the usual star. The mean density in that case turns out to be 1000 times less than the corresponding mean density of stars.

Table I contains the summary of made estimations, which shows that vacuum potential (4) allows in principle the possibility of existence of stable objects of large and

TABLE 1

Accelerations (g_{max}) on the sphere of maximum instability $(R = r_{max} = 1.5GM/c^2)$

for objects with the given mass M and corresponding mean density (see text)

M ($M_{\odot}$)	Radius	Acceleration (cm s^{-2})	Density (g cm^{-3})
1.4	R_{NS} = 10 km	$g_{NS} = 2 \times 10^{14}$	$\bar{\rho}_{NS} = 6.7 \times 10^{14}$
1.4	$R = r_{max}$ = 3.24 km	$g_{max} = 6.24 \times 10^{14}$	$\bar{\rho} \approx 15\,\rho_{NS}$
4.5	$R = r_{max}$ = 10 km	$g_{max} = 2 \times 10^{14}$	$\bar{\rho} \approx 15\,\rho_{nucl}$
5.7	R = 12.6 km	$g_{max} = 1.6 \times 10^{14}$	$\bar{\rho} = 15\,\rho_{NS}$
3 x 10^6	$R \approx R_{\odot}$	$g_{max} \sim 3 \times 10^8 \approx g_{WD}$	$5 \times 10^3 \approx 10^{-3}\rho_{WD}$
7 x 10^{10}	$R = 10^3$AU	$g_{max} < g_{star}$	$< 10^{-3}\,\rho_{star}$

superlarge mass, attaining a critical for potential (4) radius $r = r_{max} = 1.5GM/c^2$. Of course, the properties of such compact objects of GD will be different for the objects of solar and cosmological masses, which is also their difference from the black holes ofGR.

The fact must also be noted that the less and less exotic forms of matter are needed for a spherically-symmetric object with vacuum potential (4) to be made stable on the sphere with the maximum gravity acceleration or on the sphere of the maximum instability of the object with the given mass M. From the point of view of GR here it is necessary to do something impossible (absurd, paradoxical): it is necessary to take M in (16) larger and larger. In this limit, ultimately one could take the total mass of the metagalaxy.

If we summarize everything said in the last two sections, one can mark here that in GD there is a sphere of the maximum instability $r = 1.5GM/c^2$ for the given M - an analogue of the sphere of an absolute instability, i.e., Schwarzschild's sphere in GR. But in GD after the sphere $r = r_{max} = 1.5\ r_*$, the sphere $r = r_*$ of the total equilibrium vanishes which cannot be true in GR in principle.

The objects 'breaking through' to the sphere $r = r_*$, will be considered in detail in Sections 6 and 7.

5. The Motion in the Given Field (4) and the Possibility of the Periodic Pulsation of the Sphere with $R = r_*$

Now, unlike that was said in Section 3,1 shall consider some cases of the allowing for the dependence on υ^2/c^2 for the force acting on a test particle in field (4), i.e., I shall endeavour to study such situations when a test particle (or particles) performs rather slow (for the radiation could yet be not taken into consideration) radial motions in the same vacuum potential. As before, $m \ll M$, that guarantees in fact a small value of the force $F_{\max}$ at $r = 1.5\, r_*$ in comparison with F_G. Here the case of accretion on the central object under the action of the force $F_{\max}$ can be included, i.e., on the sphere of the maximum instability $r = 1.5\; r_*$, 'the rope is broken' and the fall of test particles is possible to the centre or, more exactly, to the sphere $r = r_*$ of the total equilibrium, where the force of gravity becomes zero.

Thus, let us assume that the motion of test particles in field (4) occurs in such a way that

$$\mathbf{v} = \upsilon \cdot \mathbf{r}_0 \frac{\upsilon(r)}{r}\mathbf{r} \qquad \text{and rot } \mathbf{v} = 0 \ , \tag{17}$$

i.e., υ - the value of the velocity of every particle - depends only on r, and the velocity vector is always directed along the radius-vector $\mathbf{r}$. Beside the fall to the centre, here one could also imagine, in principle, some spherically-symmetrical pulsations of the system of test particles near the sphere $r = r_*$ of the stable equilibrium (see Figure 1). In such a case of whirlwindless motion of test bodies the vector field $\mathbf{A}$ is also spiral-free,

$$\mathbf{H} = \mathrm{rot}\mathbf{A} = \mathrm{rot}\left(\frac{1}{c}\frac{\psi_{11}(r)}{\sqrt{1-\upsilon^2/c^2}}\frac{\upsilon(r)}{r}\mathbf{r}\right) = 0 \ ;$$

and here, as in Section 3, the second addend (the 'magnetic' term) in force (10) with 'uncomfortable' vector H vanishes. Thus for the force, producing the work, we obtain

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = -\frac{fm}{c}\frac{\partial A}{\partial t} - fm\nabla\varphi \tag{18}$$

where

$$\frac{\partial \mathbf{A}}{\partial t}=\frac{1}{c}(\mathbf{v}\frac{\partial}{\partial t}\frac{\psi_{11}(r)}{\sqrt{1-v^2/c^2}}+\frac{\psi_{11}(r)}{\sqrt{1-v^2/c^2}}\frac{\partial \mathbf{v}}{\partial t})$$

As far as we exclude the explicit dependence of the particle velocity on *t,* i.e., there is no 'detached' force setting the function *v(t)* in advance, then $\partial v/\partial \mathrm{t} \equiv 0$ (we keep in mind that $m \ll M$ and there is no radiation).

But then everything becomes again to look like the case of 'statics' considered in Section 3. As before, for the force acting on every test particles, only ψ_{00}-component of potential (4) is of importance: i.e., of

$$\mathbf{F}=-fm\nabla\varphi = fm\frac{\mathrm{d}}{\mathrm{d}r}\frac{\psi_{00}(r)}{\sqrt{1-v^2/c^2}}\mathbf{r}_0 \ . \tag{19}$$

In comparison with formula (12) the difference is here in the root $\sqrt{1-v^2/c^2}$ under the sign of derivative d/dr:

$$\mathbf{F}=-m\frac{\mathrm{d}}{\mathrm{d}r}\frac{\varphi_{GD}}{\sqrt{1-v^2/c^2}}\mathbf{r}_0 \ .$$

I repeat once more that all measurements are made here in the frame of reference in which the central attracting body rests *(m << M).* It concerns also the acceleration, with which test particles move and which can be obtained from the general relativistic correlation

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=\frac{\sqrt{1-v^2/c^2}}{m_i}\{\mathbf{F}-\frac{v}{c^2}(\mathbf{v}\cdot\mathbf{F})\} \ ,$$

where **F** is any force acting on the particle with the inertial mass m_i. (Here I use the notation m_i along with the notation of gravitational mass *m.)*

So far as in that case the velocity of test particles changes only in quantity and the force is directed along the velocity, we have

$$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{fm}{m_i}\sqrt{1-v^2/c^2}(\frac{\mathrm{d}}{\mathrm{d}r}\frac{\psi_{00}(r)}{\sqrt{1-v^2/c^2}}-\frac{v^2}{c^2}\frac{\mathrm{d}}{\mathrm{d}r}\frac{\psi_{00}(r)}{\sqrt{1-v^2/c^2}})$$

Thus we obtain that the centrally-symmetric motions in field (4) occur with the acceleration

$$\frac{dv}{dt}=\frac{1}{m_i}(1-\frac{v^2}{c^2})^{3/2} fm\frac{d}{dr}\frac{\psi_{00}(r)}{\sqrt{1-v^2/c^2}} \ . \tag{20}$$

Of course, the acceleration, which the particles move with, does not depend on particles mass, but using the equality $m_i/m = 1$ in (20) it is necessary to keep in mind that this equation for dυ/dt is obtained at the assumption $m_i = m << M$.

By use of (20) the notations $\beta(r) \equiv (1 - \upsilon^2/c^2)$ and $\varphi_{GD} = -f\psi_{00}$ for the acceleration, which changes the particles velocity only in quantity, one can write down in more compact form:

$$\frac{\mathrm{d}\upsilon}{\mathrm{d}t} = -\beta(r)\frac{\mathrm{d}\varphi_{GD}}{\mathrm{d}r} + \frac{1}{2}\varphi_{GD}\frac{\mathrm{d}\beta(r)}{\mathrm{d}r} \ .$$

Now we have an equation which describes the time change of the velocity field of test particles performing centrally-symmetrical movements in a given gravitational field with potential (4). As before, it is meant that the sum of the particle masses is much less than the total mass of the whole configuration. Now I shall endeavor to study the possible 'regimes of work' of formulae (19) and (20).

Rewrite once more formula (20) as

$$\frac{\mathrm{d}\upsilon}{\mathrm{d}t} = -\beta\frac{r_* c^2}{r^2}(1-\frac{r_*}{r}) - \frac{1}{2}\frac{r_* c^2}{r^2}\frac{\mathrm{d}\beta}{\mathrm{d}r}(1-\frac{1}{2}\frac{r_*}{r}) \ . \tag{20'}$$

For the force F in the same notations we obtain

$$F = -mc^2\frac{r_*}{r^2}\beta^{-1/2}(1-\frac{r_*}{r}) - \frac{1}{2}mc^2\frac{r_*}{r^2}\frac{\mathrm{d}\beta}{\mathrm{d}r}\beta^{-3/2}(1-\frac{1}{2}\frac{r_*}{r})$$

or

$$F = -\frac{GmM}{r^2}\frac{(1-r_*/r)}{\sqrt{1-\upsilon^2/c^2}} - \frac{1}{2}\frac{GmM}{r}\frac{(1-r_*/2r)}{(1-\upsilon^2/c^2)^{3/2}}\cdot\frac{d(1-\upsilon^2/c^2)}{dr} \ . \tag{18'}$$

At $\upsilon = 0$, $\beta = 1$, and dβ/dr = 0 the force and the acceleration (of the free fall) correspond to case (12) considered already. The novelty of formulae (18') and (20') is the presence of the gradient of particles velocity, i.e., of the quantity d(1 - υ^2/c^2)/dr, and of 'dangerous' denominators (1 - v^2/c^2) in the formula for the force.

Let us assume for the moment that in case in question of radial motions of test particles, their energy is, nevertheless, conserved in some way down to the 'depth' of the order of $r = r_*$ in field (4). Let a single particle falls to attracting centre from infinity with some initial velocity υ_0, which it acquired with respect to the centre in any way. So far as the energy is conserved, if we use (5) one can write the equation

$$\frac{mc^2}{\sqrt{1-\upsilon^2/c^2}}\{1-\frac{r_*}{r}(1-\frac{1}{2}\frac{r_*}{r})\} = \frac{mc^2}{\sqrt{1-\upsilon^2/c^2}\,|_{r\to\infty}} = const \ ,$$

then

$$\sqrt{1-\upsilon_0^2/c^2} = \frac{\sqrt{1-\upsilon^2/c^2}}{(1+\varphi_{GD}/c^2)}$$

It follows from this that the particle velocity (if its energy is conserved or almost conserved) will be maximum irrespective of the value of its initial velocity υ_0 in the moment when $\varphi_{\alpha\Gamma>}$ will be minimum, i.e., at $r = r_*$, where $\varphi_{GD} = - c^2/2$ (see Figure 1):

$$\upsilon^2_{max}/c^2 = 1 - ¼\,(1 - \upsilon^2_0/c^2) \quad \text{at} \quad m << M. \tag{21}$$

Thus, it turns out that even if we assumed the total conservation of the energy at the fall to the centre (which is not correct, generally speaking), then in that case also the maximum attainable velocity of the fall in potential (4) would be bounded from above by a value less than c for the particle with $m \neq 0$. The real velocity $\upsilon_{\max}$ must be still less than the value from (21) in consequence of the account for the influence of energy loss (braking) for the radiation. By this I want to emphasize the fact that the gravity force (18) is always a regular function of/- and υ. This force becomes zero on the sphere $r = r_*$ irrespective of the values of finite quantities β^{-1} and $d\beta/dr$, i.e., irrespective of the value of the particle velocity.

In the same (extreme) assumption that the energy is conserved in case of radial motions in question, one can obtain the dependences $\beta(r)$ and $d\beta(r)/dr$ at ε = const.:

$$\beta(r) = \frac{m^2c^4}{E^2}(1-\frac{r_*}{r}+\frac{1}{2}\frac{r_*^2}{r^2})^2 \ ,$$

$$\frac{d\beta(r)}{dr} = 2\frac{m^2c^4}{E^2}(1-\frac{r_*}{r}+\frac{1}{2}\frac{r_*}{r})(1-\frac{r_*}{r})\frac{r_*}{r^2} \ .$$

Thus for the acceleration and the force at the given ε = const, we shall have

$$\frac{d\upsilon}{dt} = -\frac{m^2c^6}{\varepsilon^2}\frac{r_*}{r^2}(1-\frac{r_*}{r})(1-\frac{r_*}{r}+\frac{1}{2}\frac{r_*^2}{r^2}), \text{ at } m << M\,!$$

$$F = -\varepsilon\frac{r_*}{r^2}\frac{(1-r_*/r)}{(1-r_*/r+\frac{1}{2}r_*^2/r^2)^2} \ , \tag{22}$$

from that it is seen that as a fact the denominator does not become zero at any real r.

From (21) at $\upsilon_0 = 0$ we obtain the known result (6) for the limit velocity in case of the parabolic fall to a massive centre. On the whole, a natural result is obtained also in case of the fall of an ultrarelativistic particle, i.e., in the case when at infinity the particle was flying already with the velocity almost equal to c. Then from (21) at $\upsilon_0/c \to 1$ it follows that $\upsilon_{\max} \to c$.

It can be understood also in such a way that in that case the field (4) changes weakly the initial velocity and, consequently, the energy $mc^2/\sqrt{1-\upsilon_0^2/c^2}$ (which is very big for $\upsilon_0/c \to 1$ in comparison with mc^2) of such particles. So far as on this section the question is only on the force producing the work and changing the value of particle acceleration, then from formulae (20') and (22) for dυ/dt it is seen that the cases $\upsilon \to c$ ($\beta \to 0$, dβ/d$r \to 0$) and $m \to 0$ can also be understood in a sense as the 'switching off gravitation in this limit ('chiral' one for GD). In particular, it must be true also for photons leaving the sphere $r = r_*$.

Formulae (22) can be used in all cases of centrally-symmetric motions, when the particle energy can be considered constant with a high precision. Let us suppose the test particles to be already (in some way) in bound, stable state with a definite (conserved) nonzero total energy on the sphere $r = r_*$ (see Figure 1). As it is seen from (22), the force and the acceleration become zero on this sphere. It is natural to consider small (with small small amplitude and velocity - without the radiation) deviations from the equilibrium state for such particles. For example, we can imagine a thin spherical layer of the bag matter, being situated near the sphere $r = r_*$.

Thus, we can consider harmonic oscillations of the bag surface which can arise due to the action of any (including nongravitational) disturbances near the surface $r = r_*$ of the sphere of the total equilibrium. Let us assume that the bag has here the radius almost coinciding with r_*.

I consider here the simplest case of rather slow motions when there is no radiation at all (i.e., $\upsilon^2/c^2 << 1$) and the oscillations are not damped.

From formulae (20) at $\beta = (1 - \upsilon^2/c^2) \approx 1$ and, correspondingly dβ/d$r \approx 0$, we have

$$\frac{\mathrm{d}\upsilon}{\mathrm{d}t} \approx -\frac{r_* c^2}{r^2}(1-\frac{r_*}{r}) = -c^2(1-\frac{r_*}{r})\frac{r_*}{r^2}$$

For oscillations with small amplitude near the sphere $r = r_*$ we write

$$r = r_* + \delta r = r_* \,(1 + \delta r/\, r_*) = r_* \,(1 + x)\,,$$

where $x \equiv \delta r/\, r_* << 1$. Then we can write for dυ/dt:

$$\frac{\mathrm{d}\upsilon}{\mathrm{d}t} = -c^2(1-\frac{1}{1+x})\frac{1/r_*}{(1+x)^2} = -\frac{c^2}{r_*}\frac{x}{(1+x)^3}\ .$$

Just the first term of the expansion over small deviation $x << 1$ from the equilibrium state gives

$$\frac{\mathrm{d}\upsilon}{\mathrm{d}t} \approx -\frac{c^2}{r_*}x \ ,$$

or $$\ddot{r} = -\frac{c^2}{r_*}\delta r \ , \qquad (\delta\ddot{r}) + \omega^2\delta r = 0 \ ,$$

where $$\omega \equiv \sqrt{c^2/r_*^2} = c/r_* = c/\frac{GM}{c^2} \ . \qquad (23)$$

In other words, for the case of rather slow motions in potential (4) on the whole an obvious result is obtained: a system of particles with minimum energy (on the bag surface with $R = r_*$), i.e., near the sphere of the total equilibrium $r = r_*$, can perform harmonic *($\delta r/ r_* << 1$ and $\upsilon^2/c^2 << 1$)* oscillations, pulsations with the period r_*/c.

I emphasize once more that this paper deals only with stationary states of the collapsar. The case of strong deviations *($\delta r/ r_* \sim 1$)* from the equilibrium must lead ultimately to the pulsations at considerable decrease of the total energy Mc^2 owing to the loss for the gravitational radiation, i.e., it can be already an essentially nonstationary situation.

6. Utmost Compact Objects - Collapsars and Some Properties of the Bag with the Radius $R = r_*$

In fact, at the end of the previous section the question is on the object 'breaking through' to the sphere of total equilibrium $r = r_*$, i.e., this is the object which found itself already for some reasons under its sphere of maximum instability $r = 1.5\ r_*$. It is this object, strictly speaking, which may be called the collapsar, it is, apparently, the stage of relativistic collapse - the shrinking under the sphere $r = 1.5\ r_*$. As it will be seen from the following, such an object can really be an analogue of what is called Schwarzschild black hole in GR in the sense that the collapsar of GD also possesses some universal properties which are determined ultimately by its mass *M.* But there are here considerable particularities also.

First of all I emphasize once more that tensor potential (4), one component of which is pictured in Figure 1, is the potential of the gravitational field in vacuum, created by an 'elementary' source, which at distances $r >> GM/c^2$ is perceived as a point object with the mass *M.* The field of the point is spherically symmetric by definition and potential (4) is diagonal. But if we begin to approach the central point, then for the same central symmetry of potential (4) at distances $r \sim GM/c^2$ from the centre such a 'point' or such an object turns out to consist of some more elementary, more 'fundamental' point objects with masses m_a*. This situation reminds evidently of high-energy physics - the collisions of particles with large transferred momentum. Like all consistent relativistic field theories (QED, QCD, electroweak theory) the theory of gravitational interaction also must contain the notion of the pointness of the sources of (gauge) field.

Thus, in accordance with general principles of the theory of gauge fields, the bag is a system of bound points with masses m_a* generating the gravitational field with potential (4) in vacuum surrounding the bag. The main purpose of the paper is the description of vacuum field properties of such an 'elementary' object up to $r \sim GM/c^2$. For this I departed from the fact that such an object exists, i.e., it is stable and its total energy is equal to Mc^2. In other words, the existence of the collapsar was supposed in advance (see [P1, P2]), and then, proceeding from some general axioms of theory, an attempt was made to prove this existence. Essential was to assume the existence of static solutions for the collapsar and, in particular, the existence of Newtonian limit for the field of the point with the mass M was important.

For vacuum potential (4) and in Figure 1 it is implied that if a point (a sphere) is chosen on finite distance from the centre, then it means either the point on the bag surface or one above the bag, i.e., in 'vacuum' - in the region where the energy density is not zero and there is the pressure of the gravitational field only. The most important is to remember that at $r \leq GM/c^2$, where the repulsive force begins to act on a matter point or on the points of the bag surface. At it is seen from Figure 1, this force or these forces can be rather big for the values of M of the order of (for example) one solar mass.

For at $r \leq GM/c^2$ the field cannot be already described by the static potential (4). Somewhere a situation arises when it is more and more difficult to secure the condition of the static character of the whole configuration and ultimately it becomes impossible to speak about the object with the rest mass M and Newtonian field of gravity at $r >> GM/c^2$. Let us take such an integral of the energy density of the gravitational field θ^{00}:

$$4\pi\int_r^\infty \frac{GM^2}{8\pi r^4} r^2 \mathrm{d}r = Mc^2$$

and make it equal (at some r) to the total energy of the whole configuration (the bag + the field = the collapsar). At $r = r_f \equiv r_*/2 = (GM/c^2)/2$ it turns out that for field (4) the total energy of such a spherically-symmetric object is the energy of massless gravitational field alone.

But lastly, it is difficult to coordinate the conditions (2) and (3) of the static character of the whole configuration (see formulae (51a) and (51b) in [PI]), which we used at the grounding of vacuum solution (4). Strictly speaking, such an object with the bag radius equal to r_f , will not have the rest mass because the gravitational field itself does not have it. Here, for macroscopic objects the same difficulties with totally field mass arise which were in electrodynamics with the electromagnetic mass of electron. And here, in GD, only a part of the total mass of the whole configuration can be the field one. Otherwise, 4-potential of such an object has not already its classic limit $\varphi_N = - GM/r$ and it can be said that simply at $r >> GM/c^2$ such an object does not exists.

The impossibility itself of the existence of the stable bound object with the bag radius equal to /yean be understood also in such a way that the forces of repulsion in the limit when $r \to r_f$ become so large that they do the bag to scatter in the form of gravitational waves with the total energy equal to Mc^2, i.e., this is utmost unstable situation which does not realize in nature for sure.

Absolute opposite case to such an utmost unstable situation is the case when we have the bag of the radius $R = r_* = GM/c^2$.

As before we assume the fields and matter (particles) inside the bag to be distributed and moving spherical symmetry. In that case the total energy of the whole configuration with the bag radius $R = r_*$ is divided into equal parts: a half of the total energy of the collapsar is the energy of the field alone ('the coat' of virtual gravitons), and the other half of the energy is the total energy of the bag. Thus, for the collapsar - the object (more precise, for the bag) which found itself under its sphere $r = 1.5\ r_*$ of maximum instability, an equation is true in the form

$$\tfrac{1}{2} Mc^2 \text{ (the bag with } R = GM/c^2) + \tfrac{1}{2} Mc^2 \text{ (the field)} = Mc^2 . \qquad (24)$$

How the energy ($Mc^2/2$) is divided inside the bag between bound particles with masses $m_a^* \neq 0$ and the fields, I do not know at present. But it is clear already that the bag with radius $R = r_*$ must be in equilibrium with its own gravitational field in vacuum if this field is given by potential (4).

Indeed, the forces acting on surface particles of such a bag are equal to zero. It means that the field outside the bag is as if 'switched off, upper layers of the bag do not press already on lower layers, that is quite unusual in Newtonian physics and absolutely inadmissible in GR.

Thus we can assume that in GD the physics inside the bag with $R = r_*$ is completely determined by the fields of interaction between bound ('half-naked') particles with the masses m_a^*.

Everywhere above I avoided the concretization of the bag properties. For the obtaining of the outer solution (4) I needed only spherical symmetry of the distribution and the motion of matter - the particles with masses m_a* in the bag. Now, when the properties of the vacuum field of the collapsar became somewhat more clear, I shall endeavor carefully to elucidate what is inside? Of course, inward properties of utmost compact objects with $R = r_*$ deserve a special investigation. Here I limit myself for the present to the most common concepts and semi-qualitative estimations.

If the radius of a region filled by point particles (the bag radius) is close to $R = r_*$ or even the bag radius is equal to r_*, then the integral of the energy density of only particles in the bag is anyhow less than the total energy, and for $R = r_*$ we have an inequality

$$4\pi \int_0^{R=r_*} \frac{\mu^* c^2}{\sqrt{1-\upsilon^2/c^2}} r^2 \mathrm{d}r < \frac{1}{2} Mc^2 \quad . \tag{25}$$

Even this strict inequality alone forbids all particles bound in the bag to be in rest *($\upsilon^2 \neq$* 0) in the whole volume of the bag. Only free (removed to infinity) particles can be in rest everywhere.

Really, at $\upsilon^2 \equiv 0$ we should have from (25) an inequality

$$4\pi \int_0^{r_*} \mu^* c^2 r^2 \mathrm{d}r < \frac{1}{2} Mc^2 \quad .$$

But here we enter again in conflict with mentioned conditions (2) and (3) of the static character (and in point of fact, of the existence and the stability!) of the whole configuration.

According to conditions (2) and (3) for the case in question the scalar source 'is conserved', i.e., the integration along volume for T gives always the same result *(Mc^2)* for any surfaces in vacuum, embracing the bag entirely. Thus we can write the integral

$$\int T \mathrm{d}V = 4\pi \int_0^{r=r_*} \mu^* c^2 \sqrt{1-\upsilon^2/c^2}\, r^2 \mathrm{d}r = Mc^2 \quad , \tag{26}$$

where the integration extends only along the bag volume, where $\mu^*(r) \neq 0$. And if to assume that in this volume everywhere $\upsilon^2 \equiv 0$, then we obtain an equality contradicting the inequality written above for $\mu^* c^2$.

The equality

$$4\pi \int_0^{\infty} \mu c^2 r^2 \mathrm{d}r = Mc^2$$

at $\upsilon^2 \equiv 0$ can take place, but only in limit case of particles infinitely removed from each other.

Of course, on the bag surface the particles velocity can become zero *($\upsilon^2(r)$* is equal to zero at $r = r_*$). Thus, we can really speak about some stationary (but no statical) state of the 'gas' of particles inside the bag. The particles in the bag cannot be absolutely 'cold'. They have to move, but in such a way that second powers of their velocities ('the temperature') $\upsilon^2(r)$ would depend only on r.

Of course, everything that was said is true till we do not take into account possible quantum corrections in (3) for (nongravitational) massless fields inside the bag. It is not excluded that the contribution of such effects in T should become determining at $\upsilon \to$ c in the whole volume of the bag. But close to the centre, as will be seen from the following, the particles really move with ultra-relativistic velocities the most probable.

If ε is understood (as in the manual of Landau and Lifshitz, 1973) as a general energy density of particles and fields inside the bag, then we can learn much about this quantity meaning the spherical symmetry of the problem with potential (4) for the case of the bag with $R = r_*$.

It turns out that the total energy density inside the bag with $R = r_*$ must change with r according to a law as

$$\varepsilon(r) = \frac{1}{8\pi}\frac{c^4}{G}\frac{1}{r^2} \tag{27}$$

which is determined only by fundamental constants of the relativistic theory of gravitation. Indeed, for (27) we have: (1) the integral of $e(r)$ along the sphere with $R = r_*$ gives

$$4\pi\int_0^{r_*}\frac{1}{8\pi}\frac{c^4}{G}\frac{1}{r^2}r^2\mathrm{d}r = \frac{1}{2}Mc^2$$

as it must be in accordance with Equation (24), (2) the quantity e(r) turns continuously into the energy density of the gravitational field around the bag

$$\left.\begin{aligned} \theta^{00}(r) &= \frac{1}{8\pi}\frac{GM^2}{r^4}\Big|_{r=r_*} = \frac{1}{8\pi}\frac{c^8}{G^3M^2} \\ \varepsilon(r)\Big|_{r=r_*} &= \frac{1}{8\pi}\frac{c^8}{G^3M^2} \end{aligned}\right\} \tag{28}$$

(3) the functions $\theta^{00}(r)$ and $\varepsilon(r)$ do not connect smoothly on the boundary $r = r_*$:

$$\frac{\mathrm{d}}{\mathrm{d}r}\theta^{00}(r)\Big|_{r=r_*} = -\frac{4}{8\pi}\frac{c^{10}}{G^4M^3}\ , \quad \frac{\mathrm{d}\varepsilon(r)}{\mathrm{d}r}\Big|_{r=r_*} = -\frac{2}{8\pi}\frac{c^{10}}{G^4M^3}\ ,$$

as it must be. It means to cross the boundary of two media: the gravitation (virtual gravitons) → the bag matter (with 'half-naked' particles with mass m_a*).

Now, let us assume that the total energy density on the surface of the bag with the radius $R = GM/c^2$ attains several (n = 2-10?) nuclear ones ($\rho_{nucl} \approx 2.8 \times 10^{14}$g cm^{-3}). Then the radius of such a bag is determined from (27):

$$n \times 2.8 \times 10^{14} \text{ g cm}^{-3} \text{ c}^2 = \frac{1}{8\pi}\frac{c^4}{G}\frac{1}{R_n^2}\ , \qquad R_n \approx 13.84 \text{ km}/\sqrt{n}\ . \tag{29}$$

Accordingly for the mass of the collapsar (the bag + the field) in that case we have

$$M_n = c^2R_n/G = 9.39M_\odot/\sqrt{n}.$$

For example, in the case when the surface energy density of the bag is equal to two nuclear ones, we have,

$$R_2 \approx 9.79 \text{ km} \quad \text{and} \quad M_2 \approx 6.64M_\odot$$

at the mean density of the bag matter $\bar{\rho} \approx 5.75\ \rho_{nucl}$ (of course, with the account for (24)!). If the surface density achieves four nuclear ones, then we have for the bag radius and the collapsar mass,

$$R_4 \approx 6.92 \text{ km} \quad \text{and} \quad M_4 \approx 4.69\ M_\odot\ ,$$

which will take place at the mean density of the bag matter $\bar{\rho} \approx 13\ \rho_{nucl}$ taking account of (24) again.

If (24) is not taken into account, the mean densities increase twice. From the point of view of GR such an object must collapse in the black hole long ago, and, consequently, no equation of state can be said here in principle.

From the point of view of GD, in all cases we deal with an object with finite density at nonzero mass and, as it was mentioned in Section 4, according to QCD at densities exceeding the nuclear density in several times (n = ?), the bag matter must undergo the phase transition into the quark-gluon plasma (QGP). It can be supposed that the objects with the parameters mentioned above (n = 2-4) are the gigantic macroscopic quark bags, in which even on the surface neighbouring quarks (and gluons) are compressed in such a way that they are on distances to each other less than 1 Fermi = 10^{-13} cm. I.e., even on the surface of such a bag there are no neutrons in which quarks could exist on distances greater or even equal to 1 Fermi = 1 F.

As it is seen from (27), the total density ε only increases deep into the bag, and, consequently, at the approach to its centre the neighboring quarks turn out to be compressed more and more narrowly, i.e., any two neighboring particles of QGP turn out to be at less and less (in comparison with 1 Fermi) mean distance from each other

at $r \to 0$. Thus the rule of the change of density ε in (27) does not contradict at least to the QCD notion of asymptotically free quarks and gluons in such a macroscopic QCD-bag.

And really, for the 'running' interaction constant of QCD in case of 6 varieties (flavours) of quarks one can write

$$\alpha_s \approx \frac{12\pi}{(33-2k)\ln(1F^2/\Delta l^2)} \quad , \tag{30}$$

where k is the number of 'unfrozen' flavours of quarks. Here the dimension of 1 Fermi is chosen as a value which is called the radius of the confinement of colour in QCD. For the present it is still a free parameter of the theory which must be determined from experiment. (For example, from experiments on collisions of ultrarelativistic ions.) In formula (30) the quantity Δl can be understood as the distance on which the quarks approach. Famous 'running' (decrease) of the constant α_s is obtained at unlimited approach of quarks (and gluons), i.e., at $\Delta l \to 0$.

But to regard the quantity a_s purely pragmatically, retaining for a_s only the requirement that the colour confinement in some volume, the QCD does not forbid apparently such a view to objects also. The same 'running' (decrease) of the constant of the strong (colour) interaction can be obtained by taking the value 10 km as a radius of confinement, i.e., the value of radius of macroscopic QGP-bag for the collapsar with the mass $M \sim 6.7\, M_\odot$ and the radius of the bag $R \sim 10$ km.

In that case the constant of colour interaction

$$\alpha_{sm} \approx \frac{12\pi}{(33-2k)\ln(10^2 km^2/r^2)} \tag{31}$$

remains exactly the same (large), as in (30), but at $r \approx 10$ km. Here already r can be understood as everywhere in this paper and, in particular, as in formula (27), i.e., r being the distance to the center of the bag.

As it was mentioned above, it follows from (27) that at $r \to 0$ the quarks (and the gluons) turn out to be pressed more and more tight, aiming to become ultimately asymptotically free (as it follows from QCD in that case). Thus, for such (hypothetic) model of the macroscopic QGP-bag besides formula (31) the increase of the energy density (27) towards its centre must be essential.

Specifically, in the centre of such a bag with $R = 10$ km (i.e., at $r = 1$ Fermi) the 'macroscopic' constant of colour forces will be only about 3 constants of electromagnetic interaction ($\alpha_{QED} \approx 0.0073$). For all that the density $\varepsilon(r)/c^2$ will be of the order of 5.4 x 10^{52} g cm^{-3}, and the total energy (mass) in a so small sphere (r = 1 F) will be 7 x 10^{14}g ~ $10^{-19}M$ at M~6.7$M_\odot$.

Consequently, on one hand a_{sm} really provides the confinement of colour in the bag volume with the radius ~ 10 km (i.e., as a result the bag remains white), and on the other hand in spheres (relatively to the bag centre) with decreasing radii the quarks (and the gluons) becomes more and more 'weakly interacting' particles.

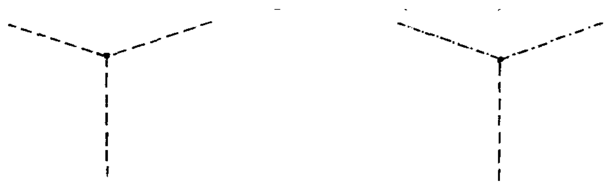
Of course, for the time being the adduced estimations of color (chromodynamic) properties of the bag must be regarded only as quality estimates. I do not insist on the absolute correctness (from the standpoint of modern problems of QCD also) of the whole identification with quarks of point particles with masses $m_a^* \neq 0$ bound in the bag and at densities $\varepsilon/c^2 > \rho_{nuci}$. But even in this paper some coincidences can be marked which allow to think that the study of the collapsar properties in GD is just 'the cross-road', where QCD and GD meet suddenly.

Specifically, it is known that the quarks masses are the smaller the tighter they are connected in a baryon. From the results of Section 2 it follows that in GD also any particle (a nucleon, for example) finding itself in a bound state on bottom of the potential well at $r = GM/c^2$, i.e., on the surface of the bag with $R = r_*$, must decrease its mass by half exactly, i.e., to change its structure! After that the gravitation is 'turned off on the surface of such a bag, it is natural to assume that inside the bag 'some' fields of interaction between bound ('half-naked') particles with masses $m_a^* \neq 0$ and at $\varepsilon(r)/c^2$ more and much more than p_{nucl}, can reduce these masses further at r-> 0, down to the chiral limit with $m_a^* \to 0$ and $\upsilon \to c$.

7. Conclusions

Through all the paper the strong gravitational field of the collapsar means concretely the case when the energy density of the gravitational field on the surface of its bag (28) coincides with the value $\varepsilon(r)$ and turns out to be equal to several nuclear densities ($>\rho_{nucl}c^2$)· As a matter of fact, apparently in that case one should speak already about the utmost strong gravitational field of the object with the (bag) radius < 10 km and the (collapsar) mass ~ 4-6.7 $M_\odot$. The precise values of the mass and the radius of such an uttermost compact object (of collapsar) depend most probably on precise values of parameters of phase QGP-transition.

So, the collapsar with the mass 4-6.7 $M_\odot$ and the bag radius <10km can be imagined as *a two-phase system.* The first phase is a bag itself- the region filled by matter (apparently, in the phase of QGP). The second phase is the gravitational field in 'vacuum' around the bag, where it (the field) interacts with itself only



and does not interact already with the bag (at least in the sense in which it was said in previous sections).

The basic properties of such an utmost compact object are presented by the following parameters:

(1) a half of the collapsar mass *($Mc^2/2$)* is contained in the bag and another half in surrounding gravitational field;

(2) the potential of the bag surface achieves its limit value equal to $\varphi_{GD} = - c^2/2$;

(3) the mass of the particle finding itself on the bag surface in bound state (at $\varphi_{GD} = - c^2/2$) is two times less than the mass of the same particle in free state (i.e., at $r \to \infty$);

(4) the energy density (of fields and particles) inside the bag is given by the dependence

$$\varepsilon(r) = \frac{1}{8\pi}\frac{c^4}{G}\frac{1}{r^2} \ .$$

Of course, all these and other properties of the collapsar are ultimately the consequences of an assumption, grounded in previous papers [P1] and [P2]:

$$\theta^{ik}{}_{(0)} = \theta^{ik}{}_{(2)}$$

I.e., in every point of 'vacuum' around the bag the energy-tension density of scalar and tensor components of gravitation are equal to each other. That is why the gravitational field of such a limit object is presented by 4-potential (4).

It is seen from Table I that it is not every object which can be in utmost bound state, i.e., under its sphere of maximum instability $r = 1.5GM/c^2$. Though the exact values of the mass and the radius of the utmost compact gravitational configuration (the collapsar) are not known yet, but one can suppose now already that the objects with the mass $<6M_\odot$ and the acceleration on the surface of the order of 10^{14}cm s^{-2} (see Table 1) are 'supermassive' neutron stars, and this are just these objects which are maximally instable relatively to the transition (the collapse) in the utmost bound state. It can be said already now that such a collapse occurs at the cost of the decrease of the total energy *(Mc^2)* of such a 'supermassive' object. It is just this collapse of the objects with masses $<6M_\odot$ and radii < 13 km which realizes apparently the phase transition when a gigantic bag is formed with quarks 'squeezed out' of nucleons (in the process of transition through the sphere $r = 1.5GM/c^2$). Such a phase transition sizes now the whole volume of the bag.

It is in this (utmost) situation that the word 'bag' approaches the sense of the notion of the bag used in QCD. In the rest of cases, and in case of big masses (especially up to cosmological ones) there is no phase transition. According to the estimates of Table I the densities in such cases can be arbitrary, moderate and small, though as earlier such objects are the most compact for their masses M. In that case they can be in a stationary state with the region dimension filled by matter greater than 2 r_*. But here one can, nevertheless, use the word 'the bag' meaning the compactness ($\theta^{00}/c^2 \approx \rho c^2$) and the gravitational connection of such bodies. Of course here there is no total cessation of gravitation action as in the case of macroscopic QCP-bag with $R = GM/c^2$.

The outer gravitation at $R > r_*$ for such bags compresses the object as in Newtonian theory, and the farther the bag dimension from the dimension of limit sphere 2 r_* , the more the deviations from dependence (27) must be for the density $\varepsilon(r)$, and there is no joint (28) on the surface of such bags already. At the increase of M the states of matter in such bags will be less exotic.

At the increase of the object mass (at $M >> 6\ M_\Theta$) there is no reason to make the object to 'break through' in the limit bound state at the given mass M. All other interaction determining the equation of state (the temperature, the pressure, the rotation momentum in hierarchical systems) in such a bag can compete with the gravitation. These objects are hardly suited to be called collapsars. Though the energy density of gravitational field near the surface of their bags can be comparable here also with the matter density in the bag (the compactness!), but an absolute value θ^{00}/c^2 is here far less than $\rho_{nucl}c^2$ at $M >> 6M_\Theta$.

Thus the distribution e(r) of the total energy (27) inside the bag with the radius $R = r_*$ (< 10 km) can be a distribution at which 'naked' and 'half-naked' point particles inside the bag are packed most closely in it. Then $R = r_* = GM/c^2$ is a radius of the most dense packing of the body rest mass with the total energy Mc^2, with the body mass approaching the value M at distances $r >> GM/c^2$.

The states with smaller dimension of the bags ($R < r_*$, but not less than $r_*/2$!) are states with the engaged 'anti-gravitation' - the repulsion. In such a state (at Figure 1 to the left from the point $r = r_*$) the bag seeks to expand. At masses $M < 6\ M_*$ the 'anti-gravitation' forces become huge, of the same order of forces which caused the collapse in the utmost bound state at $R = GM/c^2$. The bag withdrawn in that way from equilibrium must begin to pulse (with the period GM/c^3) radiating the surplus of energy in the form of scalar gravitational waves.

Summing up the whole aforesaid it is necessary to emphasize that potential (4) is a tensor potential written for arbitrary values of mass M for the field out of the bag. The notions of the mass, the radius and the properties of the utmost bound object (the collapsar proper) demand ultimately the exit beyond the limits of gravidynamics. Certainly it is necessary to account here for the information obtained at accelerators when the properties of matter in the state of QGP are studied. There are a lot of papers in astrophysics now whose authors endeavour to use QGP properties at the grounding of the possibility of the existence of quark stars in nature (within the limits of traditional approach on the basis of the Oppenheimer-Volkov equation) (Krivoruchenko, 1987; Haensel, 1987). So, the proposed paper means the motion toward almost the same aims in the investigation of the utmost compact objects properties. In particular, in gravidynamics also (like GR) the experimental (observational) investigations stay actual of the same objects - the candidates for black holes of GR.

But as it was said many times in [P1, P2], the collapsar physics is absolutely different. Ultimately the possibility of periodic oscillations (pulsations) of the QGP-bag with the period $(GM/c^2)/c$ < 10 km s^{-1} c^{-1} ≈ 3 x 10^{-5} s near the position of its total equilibrium must lead to a definite observational test of the aforesaid.

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