SCALAR GRAVITATIONAL WAVES AND OBSERVATIONAL LIMITATIONS FOR THE ENERGY-MOMENTUM TENSOR OF A GRAVITATIONAL FIELD

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(Received 9 January, 1992)

Abstract. From the point of view of a totally nonmetric model of the theory of gravitational interaction, i.e., in the bounds of a consistent dynamic description of gravitation (gravidynamics) a possibility is pointed out of additional loss of energy for the radiation of scalar gravitational waves. Such a radiation arises due to (in particular, periodic) variations, for example, spherically-symmetric pulsations, in a radiating system and is connected with time change of kinetic energy of system. The scalar gravitational 'luminosity' in gravidynamics is of the same order ($\sim G/c^5$) as the system energy loss for the radiation of 'usual' tensor gravitational waves of general relativity. Perhaps, for a binary system with a nonzero eccentricity it is necessary to account for the influence of scalar radiation on a secular variation of the companion's orbit parameters. The contribution of the scalar radiation into a total gravitational 'luminosity' of the system with a radio pulsar PSR 1913 + 16 can be of the value about 2.2% of the radiation power of the tensor gravitational waves. It can have a considerable effect at measurements of the fall rate of orbital period ($\dot{P}_b$) of the binary system, and the corresponding contribution into $\dot{P}_b$ can be equal to $\Delta\dot{P}_b \approx -0.053 \times 10^{-12}\,\mathrm{s\,s^{-1}}$.

1. Introduction

In this paper as in the very first paper (Sokolov and Baryshev, 1980) on the same topic I continue insisting on an idea that the problem of energy-momentum of gravitational field remains a central problem of gravitational physics. In other words, any theoretical scheme pretending to consistent and complete description of gravitational interaction must give concrete answers to questions about the sign, the value, the localibility of field energy and momentum in every point of space.

Proceeding from general demands being the basis of theoretical field (dynamic) description of gravitation, we grounded in the paper (Sokolov and Baryshev, 1980) the choice of an expression for θ_{ik} – the energy-momentum tensor (EMT) of gravitational field. We proceeded from the requirement (Sokolov and Baryshev, 1980; Sokolov, 1992a) that

(1) a result EMT must be symmetric $\theta_{ik} = \theta_{ki}$;

(2) it must have a trace identically equal to zero ($\eta_{ik}\theta^{ik} = 0$, where $\eta_{ik} = \mathrm{diag}(+1, -1, -1, -1)$, that is connected with zero mass of gravitons;

(3) the EMT must always give a positively defined density of gravitation field energy ($\theta_{00} \geq 0$), it concerns every component separately: both scalar field component ($\theta^{00}_{(0)} \geq 0$) and the tensor one ($\theta^{00}_{(2)} \geq 0$).

However, general principles alone do not give the firm belief that the choice of the field EMT appearance is right. Of course, crucial arguments (besides theoretical ones) could here be experiments in which the sign and the value of energy would be considerable. As it was noted to the essence of the matter in a paper by Thirring (1961), it turns out that it is in the theoretical field, dynamic interpretation of gravitational interaction where the account for the gravitation field energy continuously distributed in space around the field source gives (at least to the value order) the contribution comparable with observational one in the perihelion shift $\delta\varphi$ of the planet orbits in the Sun field. It is a so-called 'nonlinear effect' or a nonlinear contribution which is not managed to describe totally allowing only for the relativistic lag of gravitational interaction.

In other words, the Mercury's perihelion shift, in particular, requires the calculation of nonlinear corrections to the tensor 4-potential of the Sun field, the corrections arising due to allowing for the EMT of the very field in the right side of the field equations. Naturally, we have used the circumstance and have chosen in papers by Sokolov and Baryshev (1980) and Sokolov (1992a) an expression for the EMT the simplest in a sense from possible ones, satisfying the three above-mentioned conditions and which leads ultimately to the right explanation of the observed shift of the Mercury's perihelion in 100 years. I.e., we used directly the fact that in consistent theoretical field scheme describing gravitation, in gravidynamics (GD) the choice of the EMT is restricted by experiment.

The details of reasoning leading to the choice of the expression for the field EMT and a consistent allowing for nonlinear contribution into the Mercury's perihelion shift effect ($\delta\varphi = \delta\varphi_1 + \delta\varphi_2$) can be found in cited papers (Sokolov and Baryshev, 1980; Sokolov, 1992a). In particular, in Sokolov (1992a) unlike Sokolov and Baryshev (1980) the contribution into the EMT was marked out of every gravitation component – the scalar one, described by a scalar ψ, and the tensor one with a tensor potential Φ_{ik} ($\Phi_{ik}\eta^{ik} = 0$). In such a case the EMT of gravitational field θ_{ik} has the appearance

$$\theta^{ik} = \theta^{ik}_{(0)} + \theta^{ik}_{(2)}\,, \tag{1}$$

where

$$\theta^{ik}_{(0)} = a(\psi^{,i}\psi^{,k} - \tfrac{1}{4}\eta^{ik}\psi^{,m}\psi_{,m} - \tfrac{1}{2}\psi\psi^{,ik}) \tag{2}$$

and

$$\theta^{ik}_{(2)} = \tfrac{4}{3}a(\Phi_{mn}{}^{,i}\Phi^{mn,k} - \tfrac{1}{4}\eta^{ik}\Phi^{mn,l}\Phi_{mn,l} - \tfrac{1}{2}\Phi_{mn}\Phi^{mn,ik})\,; \tag{3}$$

and where the constant a is determined ultimately only by the choice of the units of potential measurement of scalar and tensor components of gravitation (see later).

It follows from (1), (2), (3) that for the gravitational field of the object of the mass M resting in the origin of coordinates in every point at the distance r from the center independently of Descartes's axes direction the field energy-tension of such a centrally-symmetric problem is given by the tensor

$$\theta^{ik} = \theta^{00}\,\mathrm{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})\,, \tag{4}$$

where

$$\theta^{00} = \frac{1}{8\pi}\,\frac{GM^2}{r^4}\,. \tag{5}$$

This energy-tension tensor for vacuum around a gravitating center corresponds to certain 'medium' which can be imagined consisting of a relativistic 'gas' of virtual (more precise, almost real at $r \gg GM/c^2$) gravitons. Here a fact is essential that the energy and the pressure of 'the gas' of scalar gravitons in every point of space around the source are equal to the energy and the pressure of 'the gas' of virtual tensor gravitons

$$\theta^{ik}_{(0)} = \theta^{ik}_{(2)} = \tfrac{1}{2}\theta^{00}\,\mathrm{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}) \,. \tag{6}$$

These are formulae (4), (5), (6) which lead ultimately to the total explanation of the shift $\delta\varphi$ of planetary orbits perihelia. A nonlinear contribution into the Mercury's perihelion shift turns out to be equal to (e is an eccentricity, a_R is a semi-major axis of the orbit)

$$\delta\varphi_2 = -\frac{\pi}{(1-e^2)}\,\frac{GM/c^2}{a_R}\,,$$

which gives in sum with the shift $\delta\varphi_1$ due to a (linear) effect of relativistic lag of gravitational interaction (see Sokolov, 1992a) the known result: namely,

$$\delta\varphi = \delta\varphi_1 + \delta\varphi_2 = \frac{6\pi}{(1-e^2)}\,\frac{GM/c^2}{a_R}\,. \tag{7}$$

I emphasize here once more that a sign of $\delta\varphi_2$ – of 'nonlinear effect' of perihelia shifts – is connected directly with the sign of the gravitational field energy θ^{00} in (5).

Of course, Equation (7) is tested also by the effect of perihelia shifts in relativistic close binaries. In particular, it was perfectly confirmed as soon it was started to observe systematically a binary with a pulsar PSR 1913 + 16 (Taylor *et al.*, 1979). But due to the fact that this formula coincides totally with the one obtained in GR long ago (in geometrodynamics) and simply due to the fact that in the case gravidynamics (GD) 'explains' the effect discovered a hundred years ago, such an observational restriction to the choice of the EMT of gravitation can seem on the face of it a certain method particularly of the same GR allowing to treat old facts somewhat otherwise. In such a situation it would be desirable to obtain absolutely new observational consequences connected with the choice of the field EMT. For all this the consequences are needed which would be connected with essential differences of GD from a geometric interpretation of gravitation in GR.

A theoretical scheme of gravidynamics developed by us in Sokolov and Baryshev (1980) and Sokolov (1992a) differs in its basic principles from GR and different theoretical variants alternative to GR by the fact that gravitation is here two components – scalar and tensor ones. For all this, for the description of interaction between field and matter (particles and fields) we need not at all an introduction of two coupling constants for every component of gravitation ψ and Φ^{ik} as it was done, for example, in a known (bimetric) theory by Brans–Dicke. In GD the coupling constant is *only one* (see Sokolov, 1990, 1992b), and nevertheless we develop a theory variant in which along with real and massless gravitons of spin 2 there exist also real massless gravitons of spin 0 – massless scalar bosons.

Thus, in GD the radiation of scalar (longitudinal) gravitational waves by a system is possible, and as far as in GR the gravitational radiation is only a tensor one, then a possibility arises of the test of a corresponding component of the tensor θ_{ik} in Equation (1) in the case, for example, of the same binary system with the pulsar PSR 1913 + 16. (θ_{00}-component of the tensor θ_{ik} is tested, as it was mentioned above, in Equation (7).)

Both tensor and scalar gravitational waves are radiated by the system simultaneously by force of the fact that both components of gravitation interact with its source with the same coupling constant f (it was emphasized specially above!). The corresponding equations of field for the scalar ψ and the tensor Φ_{ik} ($\phi_{ik}\eta^{ik} \equiv 0$) can be presented in the form

$$\Box\psi = \frac{f}{2ac^2}\,T\,, \tag{8}$$

$$\Box\Phi_{ik} = -\frac{f}{2ac^2}\,T^{(2)}_{ik}\,. \tag{9}$$

In that case – i.e., at the presence of sources on the right-hand side of Equations (8) and (9) – the scalar ψ and the tensor Φ_{ik} are connected yet by Gilbert–Lorentz's gauge condition

$$B^i \equiv \Phi^{im}{}_{,m} - \tfrac{1}{4}\psi^{,i} = 0\,,$$

which excludes a 'superfluous' vector field B^i (see in detail Sokolov, 1992b). The scalar $T = \eta_{ik}T^{ik}$ in (8) is the trace of the EMT of particles-sources of field, the tensor $T^{(2)}_{ik}$ is a massless part $T^{(2)}_{ik} = T_{ik} - \frac{1}{4}\eta_{ik}T$ of the EMT of particles constituting the radiating system.

2. Free Scalar and Tensor Gravitational Fields

To demonstrate more clearly the moments common in GR and the innovations given by GD in the case of gravitational waves, below the whole account will be constructed analogously to that was made in a known textbook (Landau and Lifshitz, 1973) in the chapter 'Gravitational Waves'. Of course, the question will be first of all on *linear* GD (Sokolov, 1990, 1992a, b).

Let us consider firstly the simplest case of the solutions of the equations of field, when there are no sources at all. I.e., this is the case of free gravitational waves and corresponding equations of free scalar and tensor fields will be

$$\Box\psi = 0\,, \tag{8$'$}$$

$$\Box\Phi_{ik} = 0\,; \tag{9$'$}$$

from which it follows that ψ and Φ_{ik}-fields propagate with the same velocity – the velocity of light c. For plane waves, propagating along the X-axis ($x^1 = x$) these equations have the form

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi=0\,,\qquad\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Phi_{ik}=0\,.\tag{*}$$

Solutions of (*) are any functions of $(t \pm x/c)$ and if we consider the case of the wave propagating in the positive direction of the X-axis then ψ and all the values of Φ_{ik} will be functions of $(t - x/c)$ only.

In the case of free fields ψ and Φ_{ik} far from sources, both components of gravitation are absolutely independent fields and it is necessary to use as condition, excluding a vector field, a gauge condition

$$\Phi^{im}{}_{,m}=0$$

(see in detail Sokolov, 1992a; Section 5). If we use these additional conditions for the functions Φ_{ik} we obtain $\dot{\Phi}^{1i}-\dot{\Phi}^{0i}=0$ (the point is the differentiation by t). And for separate components of the tensor Φ_{ik}, as in the textbook (Landau and Lifshitz, 1973) we obtain the relations

$$\Phi^{11}=\Phi^{01}\,,\qquad\Phi^{12}=\Phi^{02}\,,\qquad\Phi^{13}=\Phi^{03}\,,\qquad\Phi^{10}=\Phi^{00}\,.\tag{10}$$

But the condition $\Phi^{im}{}_{,m}=0$ allows still the change of potentials by means of (gauge) transform

$$\Phi_{ik}\to\Phi_{ik}+A_{i,k}+A_{k,i}\,,$$

where A_i can also be chosen in the form of a plane wave (4-vector A_i is the solution of the wave equation $\Box A_i = 0$) propagating in a positive direction of the X-axis: $A_i(t - x/c)$. By use of these four functions, let us make zero four-values Φ^{01}, Φ^{02}, Φ^{03}, and $\Phi^{22}+\Phi^{33}$. Then from (10) it follows that the components Φ^{11}, Φ^{12}, Φ^{13}, Φ^{00} also become zero. As a result only the values Φ^{32} and $(\Phi^{22}-\Phi^{33})$ remain nonzero: the transform with the 4-vector $A_i(t - x/c)$ does not concern these components at all.

Here (in GD) we deal from the very beginning with the traceless tensor Φ^{ik}: i.e.,

$$\eta_{ik}\Phi^{ik}=\Phi^{00}-\Phi^{11}-\Phi^{22}-\Phi^{33}=(\Phi^{22}+\Phi^{33})=0\,;$$

and correspondingly, we have not obtained anything new by manipulations with 4-vector A_i. Simply, the potentials transform of the type of $\Phi^{ik}\to\Phi^{ik}+A^{i,k}+A^{k,i}$ must be such one that not only keep unchanged the condition $\Phi^{im}{}_{,m}=0$ but also keep in force the condition of identical zeroing of the trace of the tensor Φ_{ik}. There is once more (essential!) difference from reasoning adduced in Landau and Lifshitz (1973) for the case of a metric theory which GR is.

Thus, a plane tensor gravitational wave propagating in positive direction of the X-axis is determined by two values only: $\Phi_{23}=\Phi_{23}(t-x/c)$ and $\Phi_{22}=\Phi_{22}(t-x/c)=-\Phi_{33}$, i.e., these are 'usual' transverse gravitational waves, the polarization of which is determined by a symmetric tensor of the second rank in the YZ-plane and the sum of diagonal elements is equal to zero:

$$\Phi_{ik} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Phi_{22} & \Phi_{23} \\ 0 & 0 & \Phi_{32} & \Phi_{33} \end{bmatrix} .$$

Correspondingly, from the first equation (*) it follows that it describes longitudinal gravitational waves propagating in the direction of the X-axis, if $\psi = \psi(t - x/c)$: this wave has no other component.

Below the question is on plane and monochromatic tensor and scalar gravitational waves propagating in the positive direction of the X-axis (the frequency of waves Φ_{ik} and ψ is the same): i.e.,

$$\psi = a_{(0)} \cos(\omega t - kx) , \tag{11}$$

$$\Phi_{ik} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{22} & a_{23} \\ 0 & 0 & a_{32} & a_{33} \end{bmatrix} \cos(\omega t - kx) , \tag{12}$$

where $\omega = kc$, $k = 2\pi/\lambda$.

Now, analogously to that was made in Landau and Lifshitz (1973) but for two potentials (11) and (12) we can calculate the energy fluxes in the X-axis direction for every of two types of gravitational waves. In particular, for the tensor radiation density flux according to (3) we have (to within the division on c):

$$\theta^{01}_{(2)} = a(\tfrac{4}{3}\Phi_{mn}{}^{,0}\Phi^{mn,1} - \tfrac{2}{3}\Phi_{\mathrm{mn}}\Phi^{\mathrm{mn},01}) . \tag{13}$$

For nonzero components of traceless tensor Φ_{ik} in this formula it is necessary to account for the monochromatic character of the plane wave (12) and to use the fact that period average values of $\sin^2(\omega t - kx)$ and $\cos^2(\omega t - kx)$ coincides. Then expressing again everything by Φ_{23} and Φ_{33}, we have for period average energy flux density in the X-axis direction the expression

$$\langle c\theta^{01}_{(0)} \rangle = \frac{4a}{c} [\langle \dot{\Phi}^2_{23} \rangle + \langle \dot{\Phi}^2_{33} \rangle] . \tag{14}$$

For the energy flux calculation in plane ψ-wave propagating along the X-axis we pick out from general formula (2) for the EMT of scalar field 01-component

$$\theta^{01}_{(0)} = \tfrac{3}{4}a(\tfrac{4}{3}\psi^{,0}\psi^{,1} - \tfrac{2}{3}\psi\psi^{,01}) .$$

After all derivatives have been calculated and allowance made for the fact that period average values of $\sin^2$ and $\cos^2$ coincides, and then everything expressed again by

$\psi = a_{(0)} \cos(\omega t - kx)$ we obtain as a result for the period average flux of the scalar radiation in monochromatic plane wave, i.e., (11)

$$\langle c\theta^{01}_{(0)} \rangle = \tfrac{3}{4} 2a \frac{1}{c} \langle \dot{\psi}^2 \rangle . \tag{15}$$

Plane waves (11) and (12) determine a free field or real gravitons. Here there are no field sources at all. It is a pure case when there is no admixture of virtual gravitons also. The fact that scalar and tensor gravitons can have the same origin, is fixed in (11) and (12) so far only by the fact that both wave types have the same frequency. But, generally speaking, these can be fields absolutely independent and free of any conditions.

3. The Radiation of Tensor and Scalar Gravitational Waves in Linear Gravidynamics

To emphasize the circumstance that both components of gravitation can be radiated by the same source, it is necessary to return to general equations (8) and (9), where the fields ψ and Φ_{ik} are still connected. This system of equation can be represented in an equivalent form

$$\Box(\Phi^{ik} - \tfrac{1}{4}\eta^{ik}\psi) = -\frac{f}{2ac^2} T^{ik} . \tag{16}$$

It follows from (16) that if the sources on the right-hand side are conserved (the case of linear GD (Sokolov, 1992a)), when

$$T^{ik}{}_{,k} = 0 , \tag{17}$$

then it is just the case to which the condition guaranteeing the absence of the vector field corresponds to

$$B^i \equiv \Phi^{ik}{}_{,k} - \tfrac{1}{4}\eta^{ik}\psi_{,k} = 0 . \tag{17$'$}$$

Of course, conditions (17) are true only as far as linear approximation of GD is true. Remaining in the bounds of this approximation, when conditions (17) and (17′) can be considered fulfilled with a high precision, we are going to consider here in detail the case of a weak gravitational field generated by a system of bodies moving with small velocities in comparison with c and at rather far distances from a radiating system: i.e., in the wave zone.

The solution of Equation (16) in the wave zone and at slow motions in the source have the form

$$(\Phi^{ik} - \tfrac{1}{4}\eta^{ik}\psi) = \left(-\frac{1}{4\pi}\right)\left(-\frac{f}{2ac^2}\right)\frac{1}{r}\int T^{ik}(\mathbf{r}', t - r/c)\,\mathrm{d}V' , \tag{18}$$

where $r \gg r'$, $\mathrm{d}V' = \mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'$ at $v \ll c$.

Let us obtain first the formulae for the 'usual' tensor radiation, moreover, that here the reasoning is analogous to those adduced in Landau and Lifshitz (1973). The integrals of space components of the source $T^{\alpha\beta}$ (and we shall need only them ultimately) can be expressed by the integrals containing only T^{00}-component at the use of condition (17). In linear approximation of GD (see Sokolov, 1992a) it can be assumed that the masses of the radiating system bodies do not change or the energy radiated by the system is negligible in comparison with the total energy of the system (Mc^2), i.e., the sources are conserved. As a result we have

$$\int T^{\alpha\beta}\,\mathrm{d}V' = \frac{1}{2}\left(\frac{\partial}{\partial x^0}\right)^2 \int T^{00}x^\alpha x^\beta\,\mathrm{d}V'. \tag{19}$$

For the EMT of the particles sources in (18), distributed in space with the density μ and moving with velocities v, from the general expression for T^{00} we have, in particular, $T^{00} = \mu c^2/\sqrt{1 - v^2/c^2}$. At slow motions ($v/c \ll 1$) inside the radiating system T^{00} can be presented in the form of the series

$$T^{00} = \mu c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots\right). \tag{20}$$

At rather small motions in the system, when even the second power of the ratio v/c will be small, in (20) only the first term of the serie can be kept. It corresponds to the fact that in (19) we restrict ourselves only by terms proportional to the second power of v/c: namely, as the second power of v/c always presents (implicitly) in (19) at least once time. In such a case we obtain for space components $T^{\alpha\beta}$ in (18) the expression

$$(\Phi^{ik} - \tfrac{1}{4}\eta^{ik}\psi) = \frac{f}{16\pi ac^2}\,\frac{1}{r}\,\frac{\partial^2}{\partial t^2}\int \mu x^\alpha x^\beta\,\mathrm{d}V'\,. \tag{21}$$

At far distances from the radiating system the field ultimately turns out to be free, and we deal with real gravitons. It is this field 'torn off' the sources which takes away the energy from the system. At far distances we can consider any wave to be a plane one in small scale and, consequently, using solution (21) we can calculate also the energy flux radiated by the system, for example, in the X-axis direction by means of Equation (14).

From (21) we shall need only the components Φ_{23} and $(\Phi_{22} - \Phi_{33})$. As far as in the left-hand side of (21) the scalar ψ slips out for all that, then, as in Landau and Lifshitz (1973), at the calculation of Φ_{23} and $(\Phi_{22} - \Phi_{33})$ we can use the tensor of quadrupole moment in the mass distribution of the radiating system

$$D_{\alpha\beta} = \int \mu(3x_\alpha x_\beta - r^2\delta_{\alpha\beta})\,\mathrm{d}V\,.$$

Ultimately, for unknown components of the tensor potential Φ_{ik} of the plane wave propagating in the positive direction of the X-axis we have

$$\Phi_{23} = \frac{f}{48\pi a c^2} \frac{1}{r} \dddot{D}_{23} ,$$
$$(\Phi_{22} - \Phi_{33}) = \frac{f}{48\pi a c^2} \frac{1}{r} (\dddot{D}_{22} - \dddot{D}_{33}) , \tag{22}$$

where $\Phi_{22} = -\Phi_{33}$.

Now let this wave be not only a plane one but also the monochromatic wave of type of (12). Then we can use formula (14) and for the period average energy flux in the X-axis direction we have

$$\langle c\theta^{01}_{(2)} \rangle = \frac{G}{36\pi c^5} \frac{1}{r^2} \left[\left\langle \left(\frac{\dddot{D}_{22} - \dddot{D}_{33}}{2} \right)^2 \right\rangle + \langle \dddot{D}^2_2 \rangle \right] . \tag{23}$$

In this expression all uncertainties connected with the choice of potential measurement units have disappeared, since I used a universal connection $f^2 = 16\pi a G$ (Sokolov, 1992a).

Equation (23) coincides to within the sign of averaging over monochromatic wave period with that was obtained long ago for the tensor radiation in GR. Now, carrying out exactly the same as in Landau and Lifshitz (1973) averaging over all polarizations for the time average power of monochromatic radiation of purely tensor waves (for 'tensor gravitational luminosity') we have as a result,

$$L_{(2)} = \left\langle -\frac{d\mathscr{E}}{dt} \right\rangle_{(2)} = \frac{G}{45 c^5} \langle \dddot{D}^2_{\alpha\beta} \rangle . \tag{24}$$

The fact that for an observed value $L_{(2)}$ we obtained ultimately here (in GD) the same as in GR is not surprising. The equations of field in GD (see Sokolov and Baryshev, 1980; Sokolov, 1990, 1992a) are the same as Einstein's equations, and it is the geometric interpretation of these equations in GR, which allows to speak about the tensor field only, the 'tensor' luminosity (24). In consistent dynamic interpretation of the same equations (in GD) 4-scalars ψ and T are given an absolutely definite physical sense and, correspondingly, it is necessary to allow for the energy lost by system for the scalar radiation – the scalar luminosity.

Indeed, in formula (18) the calculations were carried out for traceless tensor Φ_{ik} only. At the same time, the scalar component of the source $T = T^m_m$ and 4-scalar ψ corresponding to it, in GD must give an additional contribution into the gravitational luminosity of the system. The scalar component ψ of gravitation is the solution of Equation (8) and in the case of slow ($v^2/c^2 \ll 1$) motions inside the system (in the source) the solution of the equation in the wave zone can be obtained simply by the convolution of tensors in (18) of the form

$$\psi = -\frac{f}{8\pi a c^2} \frac{1}{r} \int T(\mathbf{r}', t - r/c)\, dV' . \tag{25}$$

For the trace of the EMT of particles, constituting the radiating system, we have $T = \mu c^2 \sqrt{1 - v^2/c^2}$, and as far as here we consider the case of slow motions then $\sqrt{1 - v^2/c^2}$ can be expanded by a small parameter $v^2/c^2 \ll 1$. Then for T we have

$$T = \mu c^2 \left(1 - \frac{1}{2} \frac{v^2}{c^2} - \frac{1}{8} \frac{v^4}{c^4} - \cdots \right). \tag{26}$$

If to neglect now here all the serie terms depending on velocities inside the system as in the case of expansion (20) for T^{00}-component, then from (25) we obtain

$$\psi \left(\frac{v^2}{c^2} = 0 \right) = -\frac{f}{8\pi a} \frac{1}{r} \int \mu(\mathbf{r}', t - r/c) \, \mathrm{d}V' .$$

This integral going over volume containing all bodies (all particles) of the system do not depend on time in the approximation which was used at the consideration of tensor radiation also. And, namely, the system mass at all motions in it: i.e., for any dependence of μ on time, is the same.

On the other hand, in the case of the tensor radiation it is the presence in (19) of the second power of time derivative which allowed us to keep in (20) the first term of expansion only. Consequently, in the same approximation, i.e., to within terms of the second rank of $v/c \ll 1$ for the case of scalar radiation in (26) it is necessary to keep also the second term of expansion into v^2/c^2 powers.

Thus it can be said that provided the total mass of all bodies of radiating system is conserved (as the mass of each of them separately) the tensor radiation arises due to the fact that in the system there is a sufficiently quick and sufficiently nonsymmetric time change of its mass distribution $\mu(r)$ (24). At the same time the scalar radiation arises due to the time *variation of kinetic energy* of particles constituting the system. As a result the scalar radiation leads to the fall of the system 'temperature'.

But if we refuse the rest mass conservation condition

$$\frac{\partial}{\partial x^k} \left(\mu \frac{\mathrm{d}x^k}{\mathrm{d}t} \right) = 0 ,$$

then we find ourselves out of bounds of linear GD approximation. On the analogy of GR it will be already the case of the strong gravitational radiation and, in particular, of the strong scalar radiation. Of course, a special consideration is needed at the considerable unconservation of the rest masses of particles interacting gravitationally. But for this problem (essentially unstable one!) also the tendency is basically clear now already: the tensor waves 'radiate' mainly the asymmetric mass distribution in the system, i.e., the system becomes more spherical as a result. And the scalar radiation takes away the energy of inner motions of the system even when the radiating system is absolutely symmetric. In particular, the compression energy at a spherically-symmetric collapse or at spherically-symmetric pulsations is taken away from the system in the form of scalar (longitudinal) gravitational waves.

In this paper we restrict ourselves so far only by the bounds of linear GD, when the body masses of a system (binary one, for example) are constant and, consequently, the total energy (Mc^2) of the system changes negligibly at the radiation of gravitational waves of both types. That is here the question is always on weak tensor and scalar gravitational waves and, correspondingly, the wave amplitudes must be sufficiently small. It is necessary to keep in mind these remarks always in every concrete case by use of the formulae obtained below.

Thus in (26) at $v^2/c^2 \ll 1$ we keep besides the first term the second term of expansion also and, substituting it in (25), we have

$$\psi = -\frac{f}{8\pi ac^2}\,\frac{1}{r}\int \mu c^2\left(1 - \frac{1}{2}\,\frac{v^2}{c^2}\right)\mathrm{d}V' \,.$$

Dropping a constant integral we obtain for the 'radiated' ψ-field to be given by

$$\psi = \frac{f}{8\pi ac^2}\,\frac{1}{r}\int \frac{\mu v^2}{c^2}\,\mathrm{d}V' \,. \tag{27}$$

Generally speaking, here it is meant that $\mu = \mu(\mathbf{r}', t - r/c)$ and $v^2 = v^2(\mathbf{r}', t - r/c)$.

Now, we are going to use below this general (in linear GD) expression for the part, changing in time, of the scalar potential, at far distance from radiating system, at small ($v^2/c^2 \ll 1$) velocities in it, in the concrete case of a binary system. As usual, we can reduce it to the case of the motion of one particle only of a reduced mass in central field. For one particle from the formula $\mu = \Sigma_a m_a \delta(\mathbf{r}' - r_a)$ we obtain $\mu = m\delta(\mathbf{r}' - r_0)$ and from (27) we have

$$\psi = \frac{f}{8\pi ac^2}\,\frac{1}{r}\,\frac{mv^2(\mathbf{r}_0, t - r/c)}{2} \,. \tag{28}$$

Thus, we have here a lagging scalar field $\psi(t - r/c)$ of a particle of mass m, situated (oscillating) somewhere near the origin of coordinates ($\mathbf{r}_0$). This is a scalar wave radiated by a mass 'tied' to the center, and the character of the dependence of ψ on time is determined by the character of oscillations of mass m near the origin of coordinates. Assuming that these oscillations occur with a certain frequency ω, we shall deal ultimately with a plane monochromatic wave of the type of (II) that allows to calculate the gravitational luminosity connected with the scalar radiation, using Equation (15) for the flux in the X-axis direction.

By use of the connection of measure constants $f^2 = 16\pi aG$, it can be obtained

$$\langle c\theta^{01}_{(0)} \rangle = \frac{3G}{8\pi c^5}\,\frac{1}{r^2}\,\langle \mathscr{E}^2_{\mathrm{kin}} \rangle \,, \tag{29}$$

for the period average flux. For the scalar waves radiation power, which is wave period average or average over period of oscillation in the source (for the 'scalar' gravitational luminosity or for mean rate of the system energy fall due to ψ-waves) it can be obtained ultimately that

$$L_{(0)} = \left\langle -\frac{\mathrm{d}\mathscr{E}}{\mathrm{d}t}\right\rangle_{(0)} = \frac{3G}{2c^5}\,\langle \dot{\mathscr{E}}^2_{\mathrm{kin}} \rangle\,. \tag{30}$$

Hereafter the rate of the system parameters change due to gravitational radiation, and we shall use sometimes the notation of the time derivative in the form of $\mathrm{d}/\mathrm{d}t$ to differ it from the time differentiation connected with an (orbital) motion in the system.

As it was expected the 'scalar' gravitational luminosity (30) turns out to be of the same order ($\approx G/c^5$) as the tensor luminosity.

4. The Scalar Radiation and the Secular Reduction of the Orbital Period of the Binary System with a Radio Pulsar PSR 1913+16

In this section the formulae obtained above will be applicated directly to the case of a close binary system with a radio pulsar PSR 1913 + 16 which is observed leading off 1974 (Taylor *et al.*, 1979) in Aquila constellation. Here I shall be interested mainly in effects connected with the energy dissipation which are described by Equations (24) and (especially) (30). All other relativistic effects tested in this natural laboratory, other things being equal (i.e., at the same parameters of the system), coincide totally for GD with that GR gives. Ultimately in GD we also deal with exactly the same formulae for the shift of the companions orbit periastra, for the gravitational red shift and for relativistic time delay of the radio signal (Sokolov, 1992a). And only the presence of a small, and apparently measurable in the nearest future, addition to the 'tensor' (i.e., connected with the tensor radiation (24)) reduction of the orbit dimension due to the scalar radiation (30) allows to hope for the perfection of the semi-phenomenology of GR in the notions of gravitational waves.

Firstly, there will be adduced the calculation with Equaton (24) for the tensor radiation in the case of the two bodies of masses m_1 and m_2 attracting themselves by Newton's law and moving along elliptic orbits around a common center of inertia (see the problem in Chapter 13 in Landau and Lifshitz, 1973). The influence of 'tensor' luminosity is accounted exactly as it was made in Peters and Mathews (1963) and adduced then in the textbook by Landau and Lifshitz (1973). In particular, for the (rotation period) average energy lost by such a system in the form of tensor radiation only, the result is obtained which is known in GR for a long time: i.e.,

$$\left\langle -\frac{\mathrm{d}\mathscr{E}}{\mathrm{d}t}\right\rangle_{(2)} = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2)}{5c^5 a_R^5}\, f(e)\,, \tag{31}$$

where a_R is a big semi-axis of orbit and where for the function depending only on the system's eccentricity e we have

$$f(e) = \left(1 + \frac{73}{24}\,e^2 + \frac{37}{96}\,e^4\right)(1 - e^2)^{-7/2}\,. \tag{32}$$

Owing to the radiation of gravitational waves by the system, a part of the total energy of orbital motion is 'radiated' away: i.e.,

$$\mathscr{E}_b = -Gm_1m_2/2a_R ;$$

and as a result the orbital dimension, its eccentricity and the orbital period must reduce. In particular, for such a value, measured directly in experiment (Taylor *et al.*, 1979), as the velocity of the secular change of orbital period $\dot{P}_b$ we obtain

$$\langle \dot{P}_b \rangle = -\frac{3}{2}\frac{P_b}{\mathscr{E}_b}\left\langle \frac{d\mathscr{E}_b}{dt} \right\rangle .$$

Moreover, allowing now only for the loss for tensor radiation – i.e., assuming that

$$\left\langle -\frac{d\mathscr{E}_b}{dt} \right\rangle = \left\langle -\frac{d\mathscr{E}}{dt} \right\rangle_{(2)} ,$$

we shall have

$$\langle \dot{P}_b \rangle_{(2)} = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P_b}\right)^{5/3} f(e)\,\frac{m_1m_2}{(m_1+m_2)^{1/3}} . \tag{33}$$

In such a form the formula for $\dot{P}_b$ is used at the interpretation of results of the observation of binary system with PSR 1913 + 16 in the latest paper by Taylor and Weisberg (1989). Here I emphasize once more that the rest of formulae for effects which are not connected with the energy dissipation into the gravitational radiation (the periastra motion, the red shift, the relativistic time lag of pulsations coming) remain in GD exactly the same as in GR. Consequently, the results of calculation of masses $(m_1 + m_2)$, m_1, of eccentricity e at a given P_b, made in Taylor and Weisberg (1989), will also be exactly the same. But at increasing precision of parameters measurement of the orbit studied since 1974 (Taylor *et al.*, 1979; Taylor and Weisberg, 1989) (see the literature in Taylor and Weisberg, 1989) of the binary system with a pulsar, the contribution into the observed $\dot{P}_b$ – secular change of the orbit period connected with the scalar gravitational luminosity (30) – must become appreciable ultimately.

For the system with a reduced mass $m = m_1m_2/(m_1 + m_2)$ the kinetic energy is equal to

$$\mathscr{E}_{\text{kin}} = \frac{mv^2}{2} = \frac{Gm_1m_2}{2a_R}\,\frac{(1 + 2e\cos\psi_R + e^2)}{1 - e^2} ,$$

where ψ_R is a polar angle in the XY-plane for the vector $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ (see Landau and Lifshitz, 1973). It is necessary to substitute this expression in (30) and to average over the rotation period. As a result we obtain for the secular fall of the orbit motion energy due to the scalar radiation only (i.e., for the scalar luminosity of the system)

$$\left\langle -\frac{d\mathscr{E}}{dt} \right\rangle_{(0)} = \frac{3}{4}\,\frac{G^4m_1^2m_2^2(m_1+m_2)}{c^5a_R^5}\, f_{(0)}(e) . \tag{34}$$

In this case the function depending only on the system eccentricity will have the form

$$f_{(0)}(e) = e^2(1 + \tfrac{1}{4}e^2)(1 - e^2)^{-7/2} . \qquad (35)$$

To within numerical coefficients and the function $f_{(0)}(e)$, Equation (34) coincides with Equation (31) for the loss of energy for tensor radiation. That is these effects are indeed of the same order. The difference between these two effects of gravitation waves radiation is the fact that since the kinetic energy of the system is constant at a zero eccentricity of the system orbit, then at $e = 0$ there is no scalar radiation also. At the same time the loss for tensor radiation is nonzero even for round ('nonpulsing') orbit. Thus, a binary system always radiates the energy connected with the change of its quadrupole moment in the mass distribution. That is the asymmetry of this distribution is 'radiated' away. Generally speaking, there is one more source which makes a definite contribution into the gravitational radiation of the binary system, it is a kinetic energy changing in time. That is it can be said that the energy of 'pulsations' connected with nonzero eccentricity of companion orbits is radiated. Of course, Equations (31) and (34) will be true until these companions approach so close that it will become impossible to use the approximation of linear GD.

Since the scalar radiation is gravitons of zero spin, they must not change the moment of binary system by definition. It can be made sure directly by calculations analogous to that was made in Landau and Lifshitz (1973). That is why an additional, due to loss (34), secular fall of the system eccentricity, like a secular fall of its dimension and the orbit motion period, depend here only on the rate of corresponding secular change of the orbit motion energy. In particular, for an additional contribution into the secular fall of orbit period we can obtain at

$$\left\langle - \frac{d\mathscr{E}_b}{dt} \right\rangle = \left\langle - \frac{d\mathscr{E}}{dt} \right\rangle_{(0)} ,$$

$$\langle \dot{P}_b \rangle_{(0)} = - \frac{9\pi}{2c^5} \left(\frac{2\pi G}{P_b} \right)^{5/3} f_{(0)}(e) \, \frac{m_1 m_2}{(m_1 + m_2)^{1/3}} , \qquad (36)$$

again in the form adopted in Taylor and Weisberg (1989).

Now let us set about numeric estimations.

According to data published in 1979 (Taylor *et al.*, 1979), binary system with radio pulsar PSR 1913 + 16 was characterized by the parameters $e = 0.617155 \pm 0.000007$; $P_b = 27906, 98172 \pm 0.00005$ s, $m_1 = (1.39 \pm 0.15) M_\odot$; $m_2 = (1.44 \pm 0.15) M_\odot$. (The periastron shift determined in Taylor *et al.* (1979) was found to be equal to 4.226 ± 0.002 grad yr^{-1}.)

The observed value of secular fall of the orbit period was determined then (in 1979) in such a way that

$$\dot{P}_b(\text{obs } 1979) = (-3.2 \pm 0.6) \times 10^{-12} \text{ s s}^{-1} . \qquad (37)$$

The secular fall of period due to tensor radiation only (33) (it is the prediction of GR also) at parameters pointed out will be

$$\langle \dot{P}_b \rangle_{(2)} = -2.405 \times 10^{-12} s\, s^{-1} . \tag{40'}$$

At the same system parameters the contribution into $\dot{P}_b$ arising due to loss for the scalar radiation (36) will be

$$\langle \dot{P}_b \rangle_{(0)} = -0.053 \times 10^{-12} \mathrm{s\, s}^{-1} . \tag{41'}$$

Thus, the contribution into $\dot{P}_b$ due to scalar loss is only 2.2% of $\langle \dot{P}_b \rangle_{(2)}$ and at the precision ($\approx 18\%$) to which the value $\dot{P}_b$ (37) was known in 1979, certainly it is impossible yet to feel this contribution.

The precision of theoretical estimations of $\langle \dot{P}_b \rangle$ is determined mainly by the precision of determination of companion masses m_1 and m_2. Therefore, a total theoretical estimate of $\dot{P}_b$ it could have been written as early as in 1979: namely,

$$\dot{P}_b(\text{theor } 1979) = -(2.458 \pm 0.008) \times 10^{-12} \mathrm{s\, s}^{-1} . \tag{38}$$

This value of $\dot{P}_b$ can be regarded as an expected one in GD and, consequently, it is necessary to seek such a precision of the value $\dot{P}_b$ measurement that for $\dot{P}_b$(obs) the third figure after point would be reliably measured. I.e., it is necessary to measure $\dot{P}_b$ to within 0.3% for to fix for sure the fact of the radiation of the scalar gravitational waves by the system PSR 1913 + 16.

As a result of systematic observations of this binary system the precision of the $\dot{P}_b$ measurement increased sharply as soon as the data were treated accumulated during about 14 years – from 1974 to 1988. In 1989, Taylor and Weisberg (1989) published an observed value of the secular fall of orbit period of binary system with the radio pulsar, which was equal to

$$\dot{P}_b(\text{obs } 1989) = (-2.427 \pm 0.026) \times 10^{-12} \mathrm{s\, s}^{-1} \tag{39}$$

and $\dot{P}_b$ determination precision became equal to 1.07%. Now the masses also of both companions of the binary system are known with more high precision: $(m_1 + m_2) = (2.82827 \pm 0.00004)\, M_\odot$; $m_2 = (1.386 \pm 0.003)\, M_\odot$; $m_1 = (1.442 \pm 0.003)\, M_\odot$. The system period remained almost the same $P_b \approx 27906.981$ s.

If – to take in Taylor and Weisberg (1989) the largest value of $e = 0.6171472$ (the $BT(I)$ model in Taylor and Weisberg, 1989) – then the only tensor radiation gives the contribution into $\dot{P}_b$ such as

$$\langle \dot{P}_b \rangle_{(2)} \approx -2.40209 \times 10^{-12} \mathrm{s\, s}^{-1} \tag{40}$$

and the scalar losses will lead to the additional contribution

$$\langle \dot{P}_b \rangle_{(0)} \approx -0.05303 \times 10^{-12} \mathrm{s\, s}^{-1} . \tag{41}$$

Now, at the precision to which the value $\dot{P}_b$(obs) is known in 1989, the contribution (41) ($\approx 2.2\%$) must be felt already. The total theoretical value of $\dot{P}_b$ (an expected value

$\dot{P}_b$ for e from the $BT(I)$ model in Taylor and Weisberg, 1989) turns out to be

$$\dot{P}_b(\text{theor } 1989) = (-2.45512 \pm 0.00021) \times 10^{-12}\,\text{s s}^{-1}\,. \tag{41a}$$

To make sure that the uncertainty to which the orbit eccentricity is known, is not so essential, nevertheless, as the precision of the determination of the system masses, we can adduce here the estimations of $\dot{P}_b$ for another model from Taylor and Weisberg (1989), for the EH model with the least $e = 0.617127$ from those which are adduced by the authors in Taylor and Weisberg (1989). As a result we obtain

$$\langle \dot{P}_b \rangle_{(2)} = -2.40167 \times 10^{-12}\,\text{s s}^{-1}\,,$$

$$\langle \dot{P}_b \rangle_{(0)} = -0.05302 \times 10^{-12}\,\text{s s}^{-1}\,.$$

Though, certainly, for used values m_1 and m_2 it would be more correct to choose the value of e corresponding to the DDGR model from Taylor and Weisberg (1989). The total theoretical estimation is almost the same as in (41a): namely,

$$\dot{P}_b(\text{theor } 1989) = (-2.45469 \pm 0.00021) \times 10^{-12}\,\text{s s}^{-1}\,. \tag{42b}$$

Note that the second figure after point did not change, and remained the same as in estimate (38) of 1979 with 'bad' values of the masses.

Thus, from the data published in 1989 the observational estimation of $\dot{P}_b$ lies within the bounds ($\pm\,\sigma$): $\dot{P}_b(\text{obs}) = -(2.401 \div 2.453) \times 10^{-12}$ s s^{-1}, and estimate (40) which does not take into account the scalar loss (the prediction of GR!) turns out to be closer to the value -2.401×10^{-12} and the theoretical estimate (41a) with a rather sure second (and third?) harmonic is closer to the value -2.453×10^{-12} for $\dot{P}_b$. Thus GR and GD are now in the same situation (for the present) in the sense of accordance with the experiment.

However, as a result of further data accumulation, the observational situation can become absolutely different to 1993. The comparison with data of 1979 (and later, see references by Taylor and Weisberg (1989)) leads to the fact that the increase of statistics must quickly reduce the uncertainty in the observed value of secular fall of orbit period. All other parameters of this remarkable system are known now already almost with the precision of celestial mechanics. Thus, in the nearest future there will be crucial or at least very solid argument in favour or against the scalar gravitational radiation.

5. Conclusions

As it was marked in Taylor and Weisberg (1989), in the bounds of GR there is no possibility to explain the contribution into $\dot{P}_b$ as large as 1% of the value (33), determined by gravitational radiation. The authors mention the account for higher order in (v/c) (see Equation (20)), the transverse motion of the system, the Galaxy acceleration, the mass-energy loss caused by pulsar spin-down. If all these possibilities change the value of $\dot{P}_b$, then the account for them can strongly influence but third and next figures after point in the estimation of $\dot{P}_b$ (see, for example, Equation (40)). The difference $(\delta \dot{P}_b)$ between the observed and theoretical values of $\dot{P}_b$ (estimated in Taylor and Weisberg (1989) by Equation (33)) which reaches the values of the order of 1%, is used by Taylor and Weisberg for the obtaining of new limit of the value $\dot{G}/G = -\delta\dot{P}_b/2P_b$. For all this the theories of the Kaluza–Klein-type and superstring one are mentioned which predict the variation of fundamental coupling constants.

In the last paper by Damour and Taylor (1991) dedicated to the binary pulsar PSR 1913 + 16 the value $\dot{P}_b$ is obtained with account for new observational data. Using directly observational $(\dot{P}_b/P_b)^{\mathrm{obs}} = -87.39 \times 10^{-18}\ \mathrm{s}^{-1}$ and theoretical $(\dot{P}_b/P_b)^{\mathrm{GR}} = -86.0923 \times 10^{-18}\ \mathrm{s}^{-1}$ values from Table I of this paper, it turns out that $\dot{P}_b^{\mathrm{obs}}/\dot{P}_b^{\mathrm{GR}} = 1.015$. It means that the measured value $\dot{P}_b$ is close to the value $\approx -2.44 \times 10^{-12}\ \mathrm{s\,s}^{-1}$. Of course, the corrections to which the paper by Damour and Taylor is dedicated, are not allowed here for. But the same corrections were not allowed for in the previous paper by Taylor and Weisberg (1989) also, where the number $\dot{P}_b^{\mathrm{obs}}/\dot{P}_b^{\mathrm{GR}} = 1.010 \pm 0.011$ was obtained. Our analysis of the results of the paper by Damour and Taylor (1991) with account for the same corrections but using only the results of 'old' observations published in the previous paper by Taylor and Weisberg (1989) does not exclude at least the fact that as the measurement precison of the value $\dot{P}_b$ increases, the difference $\delta\dot{P}_b = \dot{P}_b^{\mathrm{obs}} - \dot{P}_b^{\mathrm{GR}}$ does not decreases indeed. Though, certainly, for more definite conclusion the measurement precision should be 2–3 times greater.

Within limits of the gravitational interaction theory regarded as a total nonmetric model, accounting for the scalar gravitational radiation from the system PSR 1913 + 16 we can try to predict an addition equal to 2% to the value $\dot{P}_b$ from (33). Thus, (at least) the second figure after point in $\dot{P}_b$ will be most probably

$$\dot{P}_b = -2.45\ldots \times 10^{-12}\ \mathrm{s\,s}^{-1}\,.$$

Of course, this difference (if there it is) from GR prediction can as earlier be interpreted as a possible change of Newtonian constant ($\dot{G}/G \approx 10^{-11}\ \mathrm{yr}^{-1}$) on the time scale of the Hubble expansion. But while treating with profound respect the contribution made by the GR, I maintain that it is still premature to ignore a possibility of one or more (alternative) explanations of observational results. Moreover, as no prediction of GR verifiable experimentally has not been refuted in GD (though here new consequences are admissed), ultimate debates and covert doubts on the occasion of the energy-momentum tensor of the gravitational field ought to be tried at least to solve experimentally, observing the effects involving strong and rapidly varying gravitational fields.

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